

Unsteady Radiative Transfer in Participating Media: a Frequency-Based Numerical Approach

Mathieu Francoeur^a, Daniel R. Rousse^{b,c,*}

^a Department of mechanical engineering, University of Kentucky, 322 RGA building, Lexington, KY, 40506, USA

^b Department of mathematics, computer sciences, and engineering, Université du Québec à Rimouski – Campus Lévis,
Lévis, PQ, G6V 8R9, Canada

^c Department of applied sciences, Université du Québec à Chicoutimi, Chicoutimi, G7H 2B1, Canada

Abstract

The one-dimensional transient radiative transfer problem in the Cartesian coordinate system – an absorbing and scattering medium illuminated by a short laser pulse – is solved by the use of a Discrete Ordinates – Finite Volume method. Previous works have shown that the original numerical approach, based in the space-time domain, induces transmitted flux emerging earlier than the minimal time required by the radiation to leave the medium. Therefore, a frequency-based numerical method is formulated implemented, and validated in this paper. Results for transmittances are accurate, without physically unrealistic behaviors at early time periods. However, the frequency-dependent approach is computationally expensive; it requires approximately five times more computational time than its temporal counterpart. The next step will be devoted to the optimization of these CPU requirements.

Keywords: Radiative transfer, transient regime, frequency-based formulation, Discrete Ordinates method.

Nomenclature		Greek Symbols		
a	Anisotropy factor	-	β Extinction coefficient	m^{-1}
c	Speed of electromagnetic waves in the medium	ms^{-1}	θ Polar angle	rad
H	Heaviside function	-	μ Direction cosine	-
i	Complex constant	$(-1)^{1/2}$	τ Optical depth	-
I	Intensity	$Wm^{-2}sr^{-1}\mu m^{-1}$	Φ Scattering phase function	-
j	Spatial node	-	ω Albedo for single scattering	-
k	Frequency step	-	$\hat{\omega}$ Time-dimensionless angular frequency (pulsation)	-
m	Discrete direction	-	<i>Subscripts</i>	
n	Temporal step	-	0	Incident collimated beam
Re	Real part of a complex number	-	c	Collimated component
S_c	Source term	$Wm^{-2}\mu m^{-1}$	d	Diffuse component
t	Time	t	j	Refers to a spatial node
t^*	Dimensionless time	-	$j\pm 1/2$	Refers to the boundary of a control volume
T	Hemispherical transmittance	-	L	Refers to medium thickness
w_m	Discrete solid angle, weight of a numerical quadrature	-	p	Refers to the pulse
z	Spatial coordinate	m	<i>Exponents</i>	
			'	Other directions
			^	Frequency-dependent quantity
			k	Refers to a discrete frequency step
			m	Refers to a discrete direction
			n	Refers to a temporal step

* Corresponding author. Tel.: (418) 833-8800 Ext. 333
Fax: (418) 833-1113; E-mail: Daniel_Rousse@uqar.qc.ca

1. Introduction

The interest for transient radiative transfer has recently increased, mainly because of numerous possible uses of short pulse lasers in a wide variety of engineering and biomedical applications [1]. Optical diagnosis of absorbing and scattering media using temporal distributions of transmittance and/or reflectance remains a promising application of transient radiative transfer [1,2].

Transient effects associated with radiation heat transfer can be neglected when the time needed by the photons to leave a medium is shorter than the period of variation of the radiative source, which is the case in most engineering applications. However, a radiative flux emitted in a short time scale (from picosecond to femtosecond), such as a pulsed laser, involves non-negligible unsteady effects. Therefore, a typical transient radiative transfer problem arises when an absorbing and scattering medium is illuminated by a short laser pulse, for which the duration is of the same order of magnitude, or less, than the time needed by the radiation to leave the medium [1].

A wide variety of methods have been developed in order to solve the time-dependent radiative transfer equation. Among these methods: the Monte Carlo formulations [3-6], the classical Spherical Harmonics [1], and the modified $MP_{1/3}A$ [7], the two-flux approaches [1], the Radiation Element Method [8], the Discrete Ordinates approaches [9-15], the integral formulation [16] and, more recently, a backward method of characteristics [17].

In this paper, a Discrete Ordinates – Finite Volume (DO-FV) approach is retained, where DO and FV refer to the directional and spatial discretizations, respectively. The main drawback of a DO-FV approach, based in the time domain, is that radiative fluxes are transmitted earlier than the minimal time required by the radiation to leave the medium. This has been widely reported in the literature [9,10,13-15], and it is caused mainly by the interdependence between the spatial and temporal discretizations, and also by the numerical approximations embedded within the application of an interpolation scheme.

In order to avoid early transmitted radiation, some researchers [9,10,18] have proposed using high order spatial interpolation schemes; this has been tested by the authors in their last contribution to the ICTEA [14]. Despite the implementation of a second order Lax-Wendroff scheme, coupled with a Van Leer or a Superbee flux limiter, the physically unrealistic behavior were still present. It has been concluded that the non-physical transmitted fluxes cannot be totally avoided with an approach based in the space-time domain, but only minimized with the use of high order schemes.

On the other hand, in order to assess the use of the Diffusion approximation for the solution of transient radiation transport problems, Elaloufi et al. [19] solved the time-dependent radiative transfer equation in the space-frequency domain with a DO method. This provided the incentive for the present research work that pertains to the solution of the Radiative Transfer Equation (RTE) in the space-frequency domain.

The problem under consideration and the associated assumptions are succinctly described in the second section. The third part of the paper is devoted to the theoretical and numerical formulation of the frequency-dependent method. This approach is then applied to solve two one-dimensional transient radiative transfer

problems, and results are compared with those obtained previously by the authors from time-based techniques [13,14], and with those available in the literature.

2. Problem Description and Assumptions

Throughout this paper, the following assumptions apply: (1) the medium is a plane-parallel semi-infinite layer of thickness z_L ; (2) the layer is composed of a non-emitting, absorbing and scattering homogeneous medium, with a relative unit refractive index; (3) radiative properties are calculated at the central wavelength of the pulse's spectral bandwidth and, consequently, reference to wavelength is omitted in the notations; (4) scattering is assumed to be independent; (5) boundaries are transparent; (6) the layer is subject to a collimated short square pulse of radiation at normal incidence (the problem is azimuthally symmetric); (7) a pure transient radiative transfer regime is considered, i.e. the pulse width is less than characteristic time for the establishment of any other phenomenon [1].

More details regarding the problem description are available in reference [14].

3. Solution in the Space-Frequency Domain

3.1. Mathematical Models

Since the problem deals with collimated irradiation, the most convenient approach to fulfill this need for a mathematical solution is to consider a separate treatment of the diffuse-scattered component (I_d) of the radiative intensity. Variations of the collimated intensity (I_c) are simply described by a spatial exponential decay and a temporal term originating from the propagation of the pulse [9,13,14].

The Transient Radiative Transfer Equation (TRTE) describes spatial and temporal variations of the diffuse component of the intensity along the direction μ in a participating medium [9,13,14].

Since the objective is to solve the transient radiative transfer problem in the space-frequency domain, all temporal variables have to be transformed into frequency-dependent variables by applying a temporal Fourier Transform (FT) of the intensity [19]:

$$I(\tau, \mu, t^*) = \int_{-\infty}^{\infty} \hat{I}(\tau, \mu, \hat{\omega}) \exp(i\hat{\omega}t^*) d\hat{\omega} \quad (1)$$

where $\hat{\omega}$ is the angular frequency deriving from the dimensionless time variable $t^* = \beta ct$, and τ is the optical depth ($\tau = \beta z$). Inversely the frequency-dependent intensity can be written as the FT of the time-dependent intensity. The time-dependent Fourier analysis applied to the TRTE leads to:

$$\begin{aligned} \mu \frac{d\hat{I}_d(\tau, \mu, \hat{\omega})}{d\tau} = & -(1 + i\hat{\omega})\hat{I}_d(\tau, \mu, \hat{\omega}) \\ & + \frac{\omega}{2} \int_{-1}^1 \hat{I}_d(\tau, \mu', \hat{\omega}) \Phi(\mu', \mu) d\mu' + \hat{S}_c(\tau, \mu, \hat{\omega}) \end{aligned} \quad (2)$$

where the frequency-dependent intensity $\hat{I}_d(\tau, \mu, \hat{\omega})$ and $(1 + i\hat{\omega})$ are complex numbers. The first and second terms of the right-hand side of Eq. (2) represent respectively the attenuation by absorption and scattering and the reinforcement due to the scattering of the diffuse

part of intensity. This equation has the form of a steady-state radiative transfer equation and is called the Complex Radiative Transfer Equation (CRTE). The source term \hat{S}_c originating from the scattering of the collimated intensity is determined from a particular solution of Eq. (2) without scattering sources [19]:

$$\hat{S}_c(\tau, \mu, \hat{\omega}) = \frac{\omega}{4\pi} \hat{I}_0(\hat{\omega}) \exp[-\tau(1+i\hat{\omega})] \Phi(1, \mu) \quad (3)$$

where $\hat{I}_0(\hat{\omega})$ is obtained from the temporal Fourier analysis of the incident pulse. From the above statement, it is concluded that the solution of a transient radiative transfer problem can be obtained by solving the CRTE for each angular frequency contained in the temporal Fourier decomposition of the pulse.

3.2. Numerical Formulation

A DO-FV approach is used in order to solve the transient radiative transfer problem in the space-frequency domain. For a given angular frequency $\hat{\omega}$, a steady-state radiation transport problem is solved by calculating the frequency-dependent intensities on the J nodes and M directions of the spatial and directional discretizations, respectively. Then, integrating the CRTE over a control space $\Delta\tau w_m$ yields:

$$\hat{I}_j^{m,k} = \frac{-\mu_m (\hat{I}_{j+1/2}^{m,k} - \hat{I}_{j-1/2}^{m,k}) + \frac{\omega_j}{2} \sum_{m'=1}^M w_{m'} \hat{I}_j^{m',k} \Phi_j^{m'm} + \hat{S}_{c_j}^{m,k}}{1+i\hat{\omega}_k} \quad (4)$$

In the above, $\hat{I}_j^{m,k}$ is the frequency-dependent intensity at node j , in direction m , and at frequency k , corresponding to the discrete pulsation $\hat{\omega}_k$ for which the CRTE is solved. The variables $I_{j\pm 1/2}^{m,n}$ are the intensities at the boundaries of the control volume surrounding the node j . The subscript d , referring to the diffuse component of intensity, is omitted for more clarity.

To solve Eq. (4), one needs to relate the value of the intensity at control volume boundaries ($j \pm 1/2$) – for a specific direction and frequency – to the nodal values ($\dots, j-1, j, j+1, \dots$), by choosing an appropriate spatial interpolation scheme. In this work, a first order upwind scheme is sufficient, since the CRTE is a steady-state equation.

Eq. (4) is then solved, for a specific frequency k , for each node j and in each direction m of the spatial and angular discretizations, respectively. An iterative scheme, based upon the convergence of the source term due to the scattering of the diffuse part of intensity, is used [15].

In order to determine the frequencies for which the CRTE is solved, a temporal FT is applied on the incident square pulse:

$$I_0^k = I_0 \left(\frac{\sin \hat{\omega}_k t_p^*}{\hat{\omega}_k} - i \frac{1 - \cos \hat{\omega}_k t_p^*}{\hat{\omega}_k} \right) \quad (5)$$

The K relevant angular frequencies are selected by calculating Eq. (5) for a series of discrete pulsations $\hat{\omega}_k$. A graphical representation of the real and imaginary parts of I_0^k , as a function of $\hat{\omega}_k$, allows one to find the threshold angular frequencies, $\hat{\omega}_{-K/2}$ and $\hat{\omega}_{K/2}$, for which the CRTE has to be solved (i.e., frequencies below and above which the real and imaginary parts of I_0^k vanish).

The angular frequency discretization $\Delta\hat{\omega}_k$ is determined by performing a numerical sensitivity analysis.

Finally, the time-dependent intensities have to be recovered from the frequency-dependent intensities. This is done by applying an Inverse Fourier Transform (IFT) on the frequency-dependent intensities:

$$I_j^{m,n} = \text{Re} \left\{ 2 \sum_{k=0}^{K/2} \frac{1}{K} \exp(i\hat{\omega}_k t_n^*) \hat{I}_j^{m,k} \right\} \quad (6)$$

Eq. (6) implies that the frequency-dependent intensities can be calculated and summed only for the $K/2$ relevant angular frequencies (symmetry of the summation) in order to determine the time-dependent intensity at a given instant t_n^* . Unlike a space-time approach, the determination of the intensity at instant t_n^* doesn't require the knowledge of intensities at $0, t_1^*, t_2^*, \dots, t_{n-1}^*$.

The principal steps to solve the transient radiative transfer problem in the space-frequency domain are summarized hereafter: (1) the temporal FT of the pulse is first calculated; (2) the relevant frequencies are determined; (3) a CRTE is solved for each selected angular frequency; (4) an IFT is applied in order to derive time-dependent intensities.

4. Results and Discussions

Two typical transient radiative transfer problems are solved in this section; in both cases, the absorbing and scattering medium is illuminated by a square pulsed collimated radiation beam of unit intensity ($I_0 = 1$) and unit dimensionless duration ($t_p^* = 1$) on its boundary $\tau = 0$ [14]. When the medium is anisotropically scattering, the linear anisotropic phase function is considered ($\Phi(\mu', \mu) = 1 + a\mu\mu'$). In post-treatment, the temporal hemispherical transmittance is calculated from the time-dependent intensities [14].

The numerical codes are written in *FORTRAN* and compiled with the software *Microsoft Developer Studio – Fortran PowerStation 4* on a PC Pentium III of 600 MHz.

4.1. Optically Thick and Highly Scattering Media

In this first problem, the optical thickness of the medium is $\tau_L = 10$ while the scattering albedo is $\omega = 0.998$. The transmittance is calculated for three types of scattering media: isotropic ($a = 0$), highly forward scattering ($a = 0.9$), and highly backward scattering ($a = -0.9$). For this problem, 100 spatial nodes and 10 directions (equal weights polar quadrature) are sufficient to discretize the spatial and directional domains; no significant improvement of the results has been observed beyond these thresholds.

Also, the temporal Fourier transform of the square pulse allows the determination of pulsations for which the CRTE has to be solved (from -16π to 16π , except $\hat{\omega}_k = 0$). A preliminary parametric analysis permitted the identification of an optimal angular frequency discretization ($\Delta\hat{\omega}_k = 3 \times 10^{-4} \pi$).

Transmittances obtained by solving the CRTE with a DO-FV method are reported in Fig. 1. It is important to note that since the optical thickness (τ_L) of the slab is 10, the minimal dimensionless time required by the radiation to leave the medium (t_L^*) is also 10.

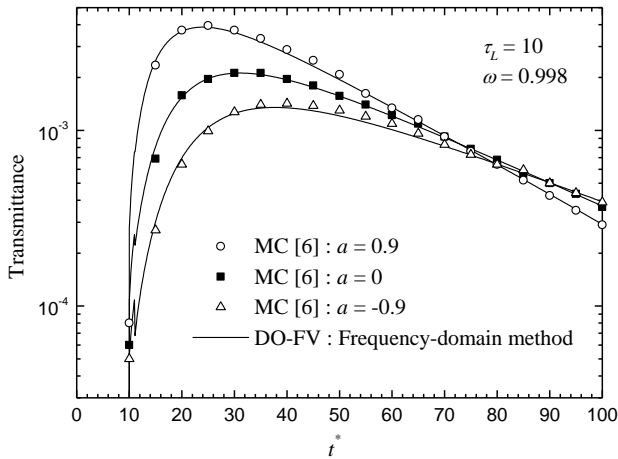


Fig. 1. Temporal distribution of hemispherical transmittance; comparison between the frequency-domain approach and a Monte Carlo formulation [6]

Figure 1 clearly indicates that for a large time scale (from 0 to 100), results from the frequency-dependent method are in good agreement with those obtained from a MC formulation, even at early time periods (i.e., near t_L^*). It is important to note here that the physical interpretation of the results is available in reference [14].

In order to analyze more precisely the relative accuracy of the frequency-domain approach, the transmittances obtained from this method are compared at early time periods with those obtained with a first order exponential scheme [13], the Van Leer flux limiter [14] and the Superbee flux limiter [14]. Only results for $a = 0.9$ are shown, since early transmitted radiation becomes more important as the medium is highly forward scattering.

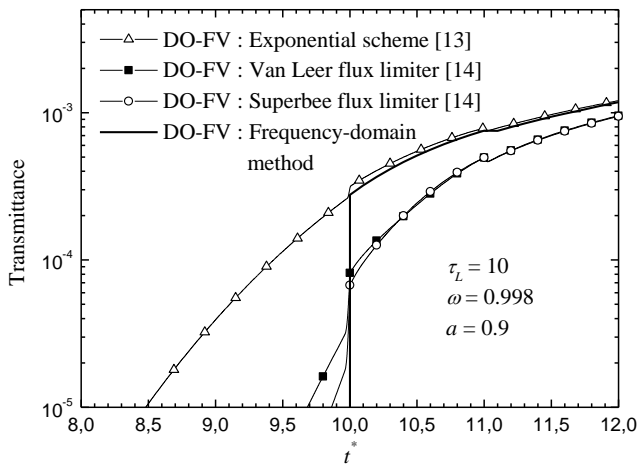


Fig. 2. Temporal distribution of hemispherical transmittance; comparison between the Van Leer flux limiter [14], the Superbee flux limiter [14], the exponential interpolation scheme [13], and the frequency-domain method at early time periods.

Figure 2 shows that no flux is transmitted before $t_L^* = 10$ only with the frequency-dependent method, since the problem associated with the interdependence between the spatial and temporal discretizations is avoided.

It can be concluded that, from a strict point of view of accuracy, the frequency-dependent approach is superior than solving directly the TRTE in the space-time domain, since the transmittance begins exactly at the dimensionless time $t^* = 10$. Even after this instant, the approaches give different results, and converged to the

same values approximately between $t^* = 10$ and $t^* \approx 12$. However, the temporal resolution of the available MC results [6] is insufficient to conclude about this divergence.

4.2. Medium of Unit Optical Thickness and Variable Scattering Albedo

In this second problem, the optical thickness of the medium is fixed at $\tau_L = 1$ while the scattering albedo ω is variable (0.25, 0.50, 0.75 and 0.90). In all cases, the medium is isotropically scattering ($a = 0$). This second test problem has been compared [15] with a Discrete Ordinates approach coupled with the Piecewise Parabolic scheme (DO-PPA) [9], which has been validated with a MC formulation [6]. The same spatial and angular discretizations used for the first problem are found sufficient.

Only a comparison between the frequency and time-dependent methods (exponential scheme [13], Van Leer flux limiter [14], and Superbee flux limiter [14]), at early time periods, is shown for this particular problem (Fig. 3). The minimal dimensionless time needed by the radiation to leave the slab of unit optical thickness is $t_L^* = 1$. Complete results are available in reference [15].

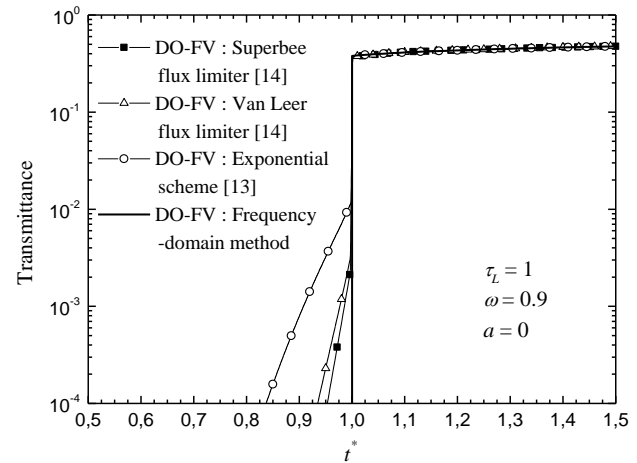


Fig. 3. Temporal distribution of hemispherical transmittance; comparison between the Van Leer flux limiter [14], the Superbee flux limiter [14], the exponential interpolation scheme [13], and the frequency-domain method at early time periods.

As for the first problem, results from the space-frequency method are the most accurate; the physics of the problem is respected, since the transmittance begins exactly at $t^* = 1$.

At the opposite of what have been observed for the first problem (Fig. 2), the Fig. 3 does not report a significant difference between the two approaches after t_L^* . This can be explain by the fact that, for a medium of unit optical thickness, the ballistic regime (collimated component) is dominant compared to the sinuous propagation regime (diffuse component), in the beginning of the temporal process. The collimated intensity does not induced early transmitted radiation, since its solution is determined exactly by a spatial exponential decay.

5. Conclusion

The one-dimensional transient radiative transfer problem for absorbing and scattering media has been

solved, in Cartesian coordinates, using a Discrete Ordinates – Finite Volume approach based in the frequency-domain.

It has been shown that the temporal distributions of transmittance obtained from the frequency-domain approach are accurate, without physically unrealistic fluxes emerging earlier than the minimal time required by the radiation to leave the medium.

However, compared to a time-dependent approach, the space-frequency method is time consuming (requires approximately, for all simulations carried out, five times more CPU time than solving directly the Transient Radiative Transfer Equation with a Superbee flux limiter). This can be explained by the fact that a large number of frequencies are needed to correctly represent the square pulse, and also, by the fact that the shift from the space-frequency to the space-time domain constitutes a supplementary computational step compared to a time-based technique.

References

- [1] Kumar S, Mitra K. Microscale aspects of thermal radiation transport and laser applications. *Advances in Heat Transfer* 1998;33:187-294.
- [2] Boulanger J, Charette A. Numerical developments for short-pulsed near infra-red laser spectroscopy. Part II: inverse treatment. *Journal of Quantitative Spectroscopy and Radiative Transfer* 2005;91:297-318.
- [3] Hsu P-f, Lu X. Reverse Monte Carlo simulations of light pulse propagation in nonhomogeneous media. Part I: theoretical developments. *Proceedings of the 4th International Symposium on Radiative Transfer*. Istanbul, Turkey, 2004.
- [4] Hsu P-f, Lu X. Reverse Monte Carlo simulations of light pulse propagation in nonhomogeneous media. Part II: numerical simulations. *Proceedings of the 4th International Symposium on Radiative Transfer*. Istanbul, Turkey, 2004.
- [5] Sawetprawichkul A, Hsu P-f, Mitra K. Parallel computing of three-Dimensional Monte Carlo simulation of transient radiative transfer in participating media. *Proceedings of the 8th AIAA/ASME Joint Thermophysics and Heat Transfer Conference*. St Louis, MO, 2002, paper no. 2002-2901.
- [6] Sawetprawichkul A, Hsu P-f, Mitra K, Sakami M. A Monte Carlo study of transient radiative transfer within the one-dimensional multi-layered slab. *Proceedings of the ASME International ME Congress and Expo*. Orlando, FL, 2004;366(1):145-153.
- [7] Wu C-Y, Ou N-R. Differential approximations for transient radiative transfer through a participating medium exposed to collimated irradiation. *Journal of Quantitative Spectroscopy and Radiative Transfer* 2002;73:111-120.
- [8] Guo Z, Kumar S. Radiation element method for transient hyperbolic radiative transfer in plane-parallel inhomogeneous media. *Numerical Heat Transfer part B* 2001;39:371-387.
- [9] Sakami M, Mitra K, Hsu, P-f. Transient radiative transfer in anisotropically scattering media using monotonicity-preserving schemes. *Proceedings of the ASME International ME Congress and Expo*. Orlando, FL, 2004;366(1):135-143.
- [10] Sakami S, Mitra K, Hsu, P-f. Analysis of light transport through two-dimensional scattering and absorbing media. *Journal of Quantitative Spectroscopy and Radiative Transfer* 2002;73:169-179.
- [11] Boulanger J, Charette A. Numerical developments for short-pulsed near infra-red laser spectroscopy. Part I: direct treatment. *Journal of Quantitative Spectroscopy and Radiative Transfer* 2005;91:189-209.
- [12] Guo Z, Kumar S. Discrete-ordinates solution of short-pulsed laser transport in two-dimensional turbid media. *Applied Optics* 2001;40(19):3156-3163.
- [13] Francoeur M, Rousse DR, Vaillon R. Analyse du transfert radiatif instationnaire en milieu semi-transparent absorbant et diffusant. *Proceedings of the 6th Colloque Interuniversitaire Franco-Québécois*. Quebec city, PQ, 2003;paper no. 08-07.
- [14] Francoeur M, Rousse DR. Analyse du transfert radiatif instationnaire en milieu semi-transparent absorbant et diffusant : utilisation de limiteurs de flux. *Proceedings of the 1st International Conference on Thermal Engineering Theory and Applications*. Beirut, Lebanon, 2004;paper no. HT5-01.
- [15] Francoeur M, Rousse DR. Numerical solutions of transient radiative transfer using time and frequency-domain approaches. *Proceedings of the ASME Summer Heat Transfer Conference*. San Francisco, CA, 2005;paper no. HT2005-72734.
- [16] Tan Z-M, Hsu P-f. An integral formulation of transient radiative transfer. *Journal of Heat Transfer* 2000;123:466-475.
- [17] Katika KM, Pilon L. Ultra-short pulsed laser transport in a multilayered turbid media. *Proceedings of the ASME International ME Congress*. Anaheim, CA, 2004; paper IMECE2004-59796.
- [18] Ayranci I, Selçuk N. MOL solution of DOM for transient radiative transfer in 3-D scattering media. *Journal of Quantitative Spectroscopy and Radiative Transfer* 2004;84:409-422.
- [19] Elaloufi R, Carminati R, Greffet JJ. Time-dependent transport through scattering media: from radiative transfer to diffusion. *Journal of Optics A: Pure and Applied Optics* 2002;4(5):S103-S108.