

## SENSITIVITY ANALYSIS APPLIED TO A SELECTION METHOD OF COGENERATION SYSTEMS BASED ON MULTIPLES CRITERIA

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### ABSTRACT

This paper studies the selection of a cogeneration system by using a multicriteria methodology involving economical, technical, thermodynamic and environmental issues. The methodology allows obtaining a ranked set of alternatives by solving a discrete optimization problem based on the Tchebycheff metric. However the parameter values and assumptions of any model are subject to changes and uncertainties. Consequently, the purpose of the paper is to investigate the impacts of some potential variations in the input variables, on the conclusions of the methodology. A study was conducted through a sensitivity analysis by means of an experimental design, consisting of the combinations of parameters which were varied from the levels at which they were set. The results shows how sensitive the solution is in the face of different parameter values as well as under what circumstances the solution would change. In addition the comparison of the sensitivity of some selecting criteria for several cogeneration sizing methods is presented.

### INTRODUCTION

Nowadays, literature regarding design and selection methodologies for cogeneration systems is widely accessible from the research community. During the last two decades, there have been many attempts at introducing more complex arrangements to improve the performances of thermal-power systems. Amongst which the favoured option is cogeneration also called CHP (Combined heat and power): this is the simultaneous production of electrical and useful thermal energy in the same power plant [1]. The technique can increase energy use efficiencies significantly, and thus reduce the net energy

consumption in almost all situations where both heat and power are required. In order to utilize CHP system's high economical and energy-saving potentials the system planning, especially the capacity of prime movers, is very important [2].

### NOMENCLATURE

$Z$	Optimality criterion
$V_p^w$	Weighted <i>Tchebycheff metric</i>
$Z^{id}$	Ideal value of a given Optimality criterion
$Z^{max}$	Maximum value of a given Optimality criterion
$U$	Utility function
$\lambda$	Weight of each criterion defined by the Decision maker
$w'$	Weight of each criterion defined by using the Entropy method
$\beta_0$	Constant value
$\beta_{ij}$	Standardized regression coefficients
$x_j$	Independent input parameter
$\hat{Y}$	Dependent output parameter
$k$	Number of experimental factors
Subscripts	
$p, n$	Dimensional space
$i, j, f$	Indices of variables
$l, m, r$	Indices of variables

Cardona et al in [3] presented a methodology for sizing a trigeneration plant which started from the results on energy consumption research in the hotel's sector, and in particular from the complete data on thermal and cooling consumption in several European hotels. More recently researchers are interested in the optimization of plant lay out and the real time optimization of operation strategies for existing plants [4]. In addition, Burer et al. showed that modelling and optimization of integrated energy systems regarding technical, economic and environmental issues could be undertaken by using the so-

called ‘environomic’ formulation, which includes all criteria within a single objective aggregated function [5]

However the parameter values and assumptions of any model are subject to changes and uncertainties. If parameters are uncertain, statistical analysis can give information concerning the sensitivity and robustness of the selection methodology studied.

As was observed by Breesch in Ref. [6], when the input parameters  $x_j$  are independent, the standardized regression coefficients (SRC) provide a measure of variable importance since SRC measures the effect of the variation of an input parameter  $x_j$  with a fixed fraction of its standard deviation on the variation of the output  $Y$ , while all other input parameters equalize their expected value. Regression techniques allow the evaluation of sensitivity of individual model inputs, taking into account the simultaneous impact of other model inputs on the result.

Sensitivity analysis methods (SA) have been applied in various fields including complex engineering systems, economics, physics, social sciences and others [7-10]. Moreover it can also be used to provide insight into the robustness of model results when making decisions [11]. Consequently, the purpose of the paper is to investigate the impacts of some potential variations in the input variables, on the conclusions of the selection methodology. In this study a sensitivity analysis by means of an experimental design, consisting of the combinations of parameters which were varied from the levels at which they were set, was conducted. Sensitivity analysis, roughly defined, is the investigation of these potential variations and their impacts on the conclusions of the model.

In this paper the selection of cogeneration systems by using a multicriteria methodology, which involves economical, technical, thermodynamic and environmental issues is studied. The methodology allows obtaining a ranked set of alternatives by solving a discrete optimization problem based on the Tchebycheff metric.

The equipment sizing and the energy performance characteristics of cogeneration systems are strongly determined by system configuration and operational strategy. The following five cases (alternatives) were considered: sizing following the traditional thermal demand management, CHDM, (A1); sizing following the power demand management, CPDM, (A2); sizing by using the maximum power demand, CogP, (A3); sizing following the cooling and heating demand management, CCHP, (A4) and sizing following the power, cooling and heating demand management, CChM, (A5) namely *integrated demands management* method.

The results show how sensitive is the solution is in the face of different parameter values as well as under what circumstances the solution would change. In addition the comparison of the sensitivity of some selecting criteria for several cogeneration sizing methods is presented.

## MULTICRITERIA SELECTING PROBLEM

The present paper is a continuation of previous work presented by the authors in Ref. [12,13]. The selecting problem for a cogeneration system, from several points of view: economical, technical, thermodynamic and environmental issues can be very complex. The most important steps in defining and solving the selecting problem consists of identifying, structuring and providing a methodology which allows taking into consideration all the aspect involved. Accordingly, the problem can be formulated as follows: Decide the best size, investments and operation strategies in order to cover the energy service demands considering different technologies, configuration, management scenarios, operational strategies as well as constrictions and requirements. Consequently we propose to separate the problem into a sizing and operation problem and an investment and selection problem. The whole problem can be seen as a multi-criteria problem that can be solved, for instance, using interactive and computational techniques.

The optimization process expresses the relationship between the design parameters and the overall system performance. It is expressed in terms of the utility function ( $U$ ). This structure is assumed to be disposed to discrete optimization and, therefore, formulated in terms of a set of input system design parameters ( $a_i, u_i, d_i$ ) and a set of output system performance measures  $\psi_m$  (efficiency indicators). Maximizing or minimizing levels of such indicators is, therefore, translated into design and improvement objectives. Multi-criteria approach is the search for the problem’s optimal solution, taking into account the multiple objectives that form it. This gives the problem a vector character. Consequently, considering evaluating efficiency indicators counted  $m$  and evaluating objects (alternatives) counted  $n$ , the original indicators values can be defined as a decision matrix  $\Psi = (\Psi_{ij})_{mn}$ .

Following, a concise discussion concerning the different efficiency indicators or criteria that were included in the cogeneration selecting problem is presented. A most frequently used economical criterion to compare different investment possibilities, the Net Present Value, was used. Another economical criterion to a better understanding the evaluation of a project, the investment payback period was considered. The Fuel Energy Saving Ratio, which measures the fuel savings directly in a CHP system, was chosen. Two major criteria for the environmental assessment are considered. The amounts of CO<sub>2</sub> not sent to the atmosphere, as well as, an Environmental Cost–Benefit criterion to evaluate the local emissions impact of each alternative were included.

The objective function of the optimal selecting problem is the minimization of the vector space function. The image function of this vector space can be seen as a utility function [12-14]. If an appropriate utility result of each possible solution is obtained, then the most desired plan of action is given for the alternative with the best expected utility.

$$\text{Min} \left\{ \left[ Z_1(\psi_1), Z_2(\psi_2), \dots, Z_m(\psi_m) \right] \right\} \quad (1)$$

Subject to:

$$g_j(d_r, u_k) \geq b_f$$

The theory offers us several frameworks in order to obtain a better approximation of the reality, see Ref. [15,16]. One possible way is assuming that the decision makers always want to obtain the closest alternative to the ideal function. If the minimum value of each criterion is obtained without taking into account the consequence in other criteria, an ideal vector space and consequently an ideal utility function can also be defined.

$$U^{id} = U_{(Z_1^{id}, \dots, Z_m^{id})} \quad (2)$$

The process consists of calculating the distance or metric between the utility function of each alternative and the ideal utility function. We use the weighted *Tchebycheff metric* ( $p=\infty$ ) based on the description in Ref. [12-16]

$$V_{p,w}^i = \left[ \sum_{j=1}^m \left( w_j^* \frac{|Z_j - Z_j^{id}|}{|Z_j^{mx} - Z_j^{id}|} \right)^p \right]^{1/p} \quad (3)$$

$$p = \infty \quad \{Z_j, Z_j^{mx}, Z_j^{id} \in R^p\}$$

The effectiveness of the proposed methodology is further illustrated by mean of a numerical example about a diesel engine cogeneration plant for a hotel in tropical conditions in previous work of the authors [12,16]. In the study case, steam is used for cooking and supplying domestic hot water, while an electric chiller is used to cover the cooling demand. A probable preferences system, criteria weight, from a given decision-maker was defined. Having defined the preference system, the *Tchebycheff metric* for each alternative was calculated. See figure 1. The alternative with the shortest value of this metric will be the best one, since it means that it is the closest one to the ideal function. In the current case of study the alternative A5 results in the most favourable solution. Once the best alternative is defined, the equipment capacities can be estimated on the basis of the selected strategy

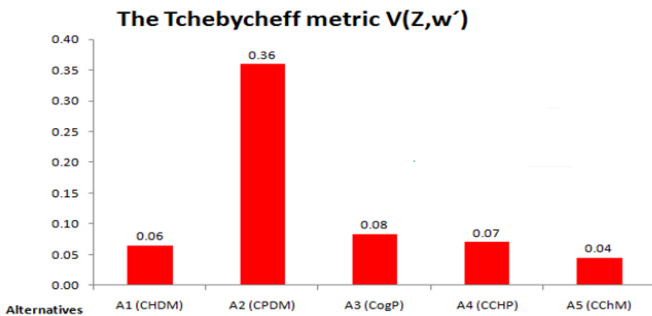


Figure. 1. The *Tchebycheff* metric for each alternative.

## SENSITIVITY ANALYSIS

As was previously mentioned, the parameter values and assumptions of any model are subject to changes and uncertainties. Therefore if parameters are uncertain, sensitivity analysis can give crucial information about the selecting process. In principle, sensitivity analysis is a simple idea: change the model and observe its behaviour. In practice there are many different possible ways of varying and observing the model. Usually, the approach is to change the value of a numerical parameter through several levels. In this study, a factorial analysis  $I^k$  with 6 factors at two levels (low and high) was carried out. An experimental design consisting of the combinations of parameters which were varied in the levels at which they were set, was conducted. The rank obtained for each alternative in every one of the sixty four different cases resulting from the factorial analysis were storied in a vector variable (from 1 to 5) namely, Ranking\_A. We make the assumption that the alternative with the lower value of variable Ranking\_A would be the selected solution in a given case. Figure 2 denotes that in most of the cases the alternative five have the best ranking position, as a result of getting the minimum values of the *Tchebycheff* metric.

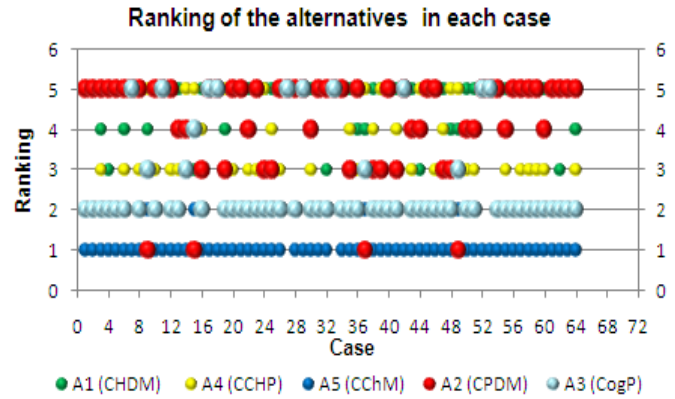


Figure 2 Ranking of the alternative in each case

For each method to size and to operate the cogeneration system, the SRCs were applied to determine the sensitivity of the Ranking\_A and consequently the influence in the selection process. The statistical model upon which the analysis of screening designs is based expresses the response variable  $Y$  as a linear function of: the experimental factors, interactions between the factors, and an error term, which can be expressed as:

$$\hat{Y} = \beta_0 + \sum_{i=1}^k \beta_i x_i + \sum_{i=1}^k \sum_{j=i+1}^k \beta_{ij} x_i x_j + \varepsilon \quad (4)$$

The experimental error  $\varepsilon$  is typically assumed to follow a normal distribution with a mean of 0 and a standard deviation equal to  $\sigma$ . Consequently, all the input parameters of the simulation model are assumed to be normally distributed. The levels selected for each parameter included a likely range of

possible outcomes for each variable. Table 1 shows the low and high level set for each parameter.

Table 1. The low and high level set for each parameter.

Parameters	Name	Design case	High level	Low level
Electric efficiency of cogeneration	EffeC	0.33	0.37	0.29
COPE of electric chiller	COPe	2.95	3.30	2.60
COP of absorption chiller	COPa	0.7	0.78	0.62
Fuel oil cost [€/ton]	HFcost	308	355	262
Cogeneration plant cost [€/kWe]	Ccost	1000	1150	850
Subsidy of CHP plant cost [%]	Subsidy	0	20	0

For this study, the analysis was carried out with the help of a statistic computer program Statgraphics Plus which is a comprehensive package designed for interactive statistical data analysis. The results obtained from the software displayed a regression model which is fitted to the data. These models were used to predict the response at specified values of the experimental factors. Commonly, in order to simplify the interpretation of screening designs, the model is expressed in terms of “effects”. For the response surface designs the “Pareto Charts” displays each of the estimated effects in decreasing order of magnitude. The length of each bar is proportional to the standardized effect, which is the estimated effect divided by its standard error. Figure 3 and figure 4 shows the Pareto Charts for alternatives A1, A4 and A5.

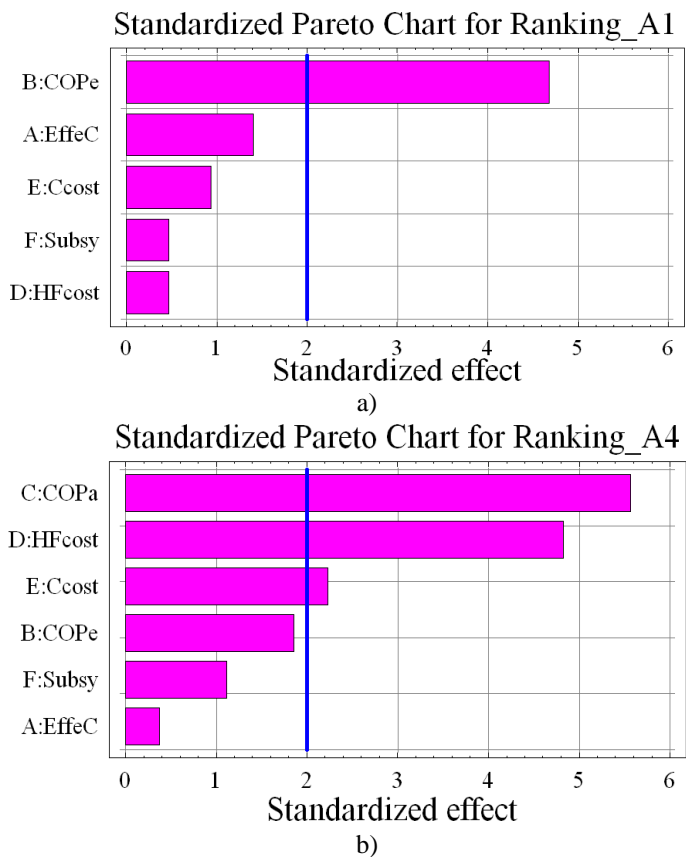


Figure 3. The Pareto Chart of alternatives A1 and A4.

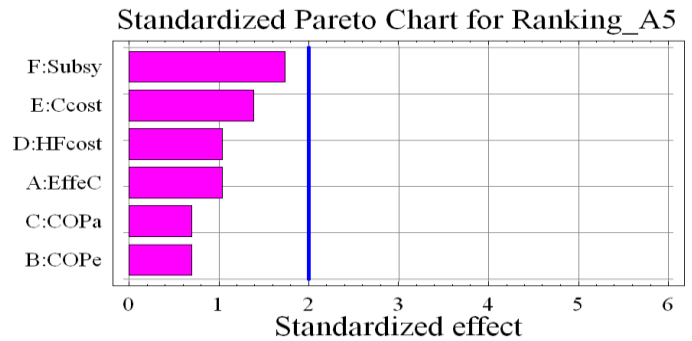


Figure 4. The Pareto Chart of alternative A5.

In the above figures any bars which extend beyond the line correspond to effects which are statistically significant at the 95% confidence level. Note that in the charts interactions were excluded since they were not statistically significant. In graphic a) only five variables are considered, since in this alternative, A1, the cogeneration system only produces heat and power, hence the absorption chiller does not exist and so the COPa was not considered. Moreover, the charts a) and b), for instance, traditional cogeneration sizing method and traditional trigeneration sizing method, (A1 and A4, respectively), denotes that in these two alternatives always there are one or more variables with high statistical significance in the behavior of the selection process. In the case of alternative A1, the *Coefficient of performance of the electric chiller* is the most important factor bringing about the high sensitivity of the system in the face of any change of this parameter. On the other hand the *Coefficient of performance for absorption chiller* and the *Fuel cost* are the most important variables when the trigeneration system is selected by using the traditional sizing method for trigeneration systems. It is noticeable that for the case of alternative A5, when the trigeneration system is selected following the power, cooling and heating demand management by using the CChM sizing method, the six variables analyzed are not statistically significant.

In order to compare the influence of the analyzed variables for all alternatives, figure 5 display the estimated effects for each alternative. It is clear that the most robust method for selecting trigeneration system is the CChM sizing method since it is the only method where the six variables analyzed are not statistically significant. As a result of achieve a better compromise between the selection criteria.

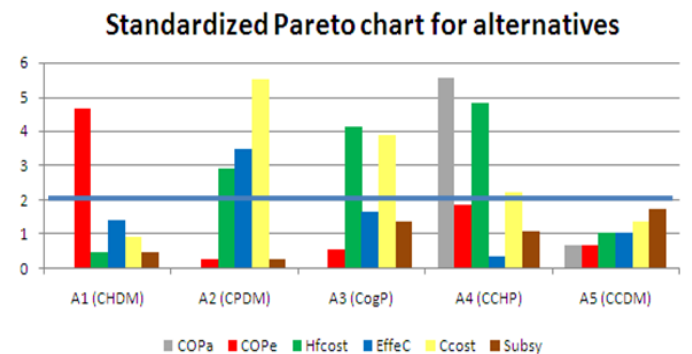
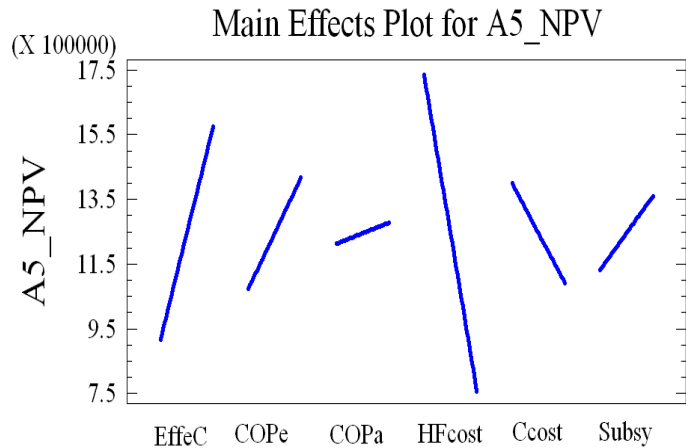


Figure 5. Estimated effects in Ranking\_A for each alternative.

Following a more detailed analysis of the sensitivity for alternative A5, the *integrated demands management* method (CChM) is presented. The SRCs were applied to determine the sensitivity of the NPV when the *integrated demands management* method was used to size and to operate the cogeneration system. The main effect of factor  $j$  can be defined as the change in the response variable  $Y$  when  $X_j$  is changed from its low level to its high level, with all other factors being held constant midway between their lows and their highs. Figure 6 shows the main effects plot for the present response surface designs.



**Figure 6.** Main effect of technical and economical parameters in the Net present value.

In this section, two main groups of such parameters can be defined. The first group deals with rather technical parameters, such as *Electrical efficiency of cogeneration*, the *Coefficient of performance of the electric chiller* and the *Coefficient of performance of the absorption chiller*. The second group consists of rather market related parameters, such as, *Fuel cost*, *Initial costs* and *Subsidy for the initial cost*.

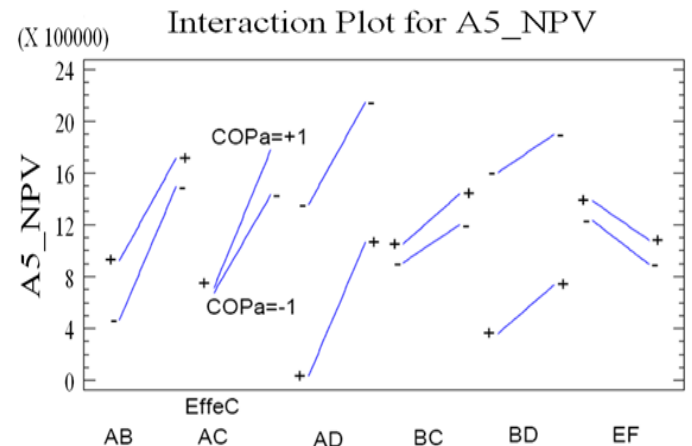
The above graphic show clearly that in the first group, the *Electric efficiency of cogeneration* is the most important parameter. To show the impact of changing technical parameters, values of these factors were set individually from 90 to 110 per cent of the values in their base case, and NPV was calculated. The *Electric efficiency of cogeneration* has the largest influence on the economics of the plant, with almost a 13% variation in the NPV. The same relative change of the other two technical parameters definitely has less influence on NPV, consisting of approximately 3.5%, caused by the variation of the *Coefficient of performance of the electric chiller* and 1.5% from the variation of the *Coefficient of performance of the absorption chiller*.

On the other hand, in the second group, varying the *Fuel cost* has the strongest influence on the economics of the plant. In this study case, raising the power price towards 110% of the base case values will increase 45% the NPV. The impact of a variation of the *Fuel cost* happens to be considerably larger than the impact of varying the *Initial costs*, as well as, a

variation of the *Subsidy for the initial cost* only influences the NPV to a minor extent.

Another important aspect consists on evaluating the interactions existing amongst the experimental factors. To investigate about the factors interaction, the effect graphic should be produced for each pair of factors. In figure 7 a pair of lines was plotted for each interaction, corresponding to the predicted response when one factor is varied from its low value to its high value, at each level of the other factor. All factors not involved in the interaction are held at their central value.

The predicted response for each combination of the low and high levels of two factors is displayed at the end of each line segment. If two factors do not interact, the effect of one factor will not depend upon the level of the other and the two lines in the interaction plot will be approximately parallel. If the factors interact, as it is the case of AC in the figure bellow, the lines will not be parallel and may even cross.

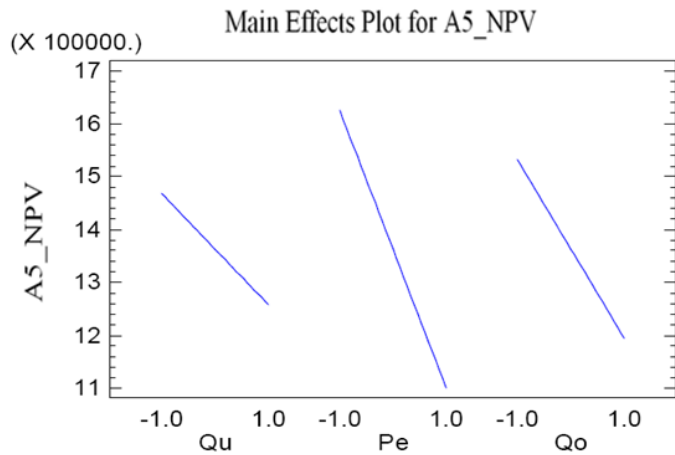


**Figure 7** Interaction effect of technical and economical parameters in the Net present value.

As one can see in the present sample, in most cases the line are close to parallel showing that the effect of one factor will not depend upon the level of the other. However, the plot above shows that the *Coefficient of performance of the absorption chiller* has little effect on the response at a low level of the *Electric efficiency of cogeneration*, while, it has a large effect at the high level of the *Electric efficiency of cogeneration*.

Finally the possibility of changing significantly the energy demands (power, heat and cooling) after the design of the cogeneration plant is fixed was considered. Consequently, a similar approach was used and a  $2^3$  factorial design was conducted. As a result the impact on the economic feasibility of the cogeneration plant due to the variation of the energy demands was clarified. The NPV of the cogeneration plant was calculated for a variation of the energy demands between 80% and 120% of the base case value. Results are shown in figure 8 presenting “The main effects plot” for the given response surface designs.

It is noticeable that the power demand is the most important factor while all the interactions are not statistically significant. This factor has the largest influence on the economics of the plant, with almost 33% of variation in the NPV.



**Figure 8** Main effect of energy demands variation in the Net present value.

The similar relative change of the other two factors certainly has less influence on NPV, consisting of 15% caused by the variation of the *Cooling demand* and 8% from the variation of the *heat demand*. It is clearly shown that the variation of the energy demands has a significant influence on the economics of the plant. This stresses the importance of a suitable design of the cogeneration plant and a correct analysis of the future energy demands at the site.

## CONCLUSIONS

The impacts of some potential variations in the input variables, on the conclusions of a multicriteria selecting methodology for cogeneration systems were investigated. A sensitivity analysis by means of an experimental design, consisting of the combinations of parameters which were varied in the levels at which they were set, was conducted.

The importance of some technical parameters, like electric efficiency of cogeneration system and some economic parameter like the fuel cost, was shown. Additionally, the interactions existing amongst the experimental factors were evaluated. This analysis allow identifying that the *Coefficient of performance of the absorption chiller* has little effect on the response at a low level of *Electric efficiency of cogeneration*, while, it has a large effect at the high level of *Electric efficiency of cogeneration*.

It can be concluded that for cogeneration plants working under tropical conditions, *integrated demands management*, CChM, is more robust than the traditional methods to size and to operate cogeneration and trigeneration systems. The sensitivity analysis shows that the CChM was the only method where the influence of the six parameters analyzed was not statistically significant. Finally, the influence on the economics of the plant of changing the energy demands was presented.

The importance of a suitable design of the cogeneration plant and a correct analysis of the future energy demands at the site was demonstrated. The results shows how sensible the solution is in the face of different parameter values and under what circumstances the solution would change.

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