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Numerical study of the Discontinuous Galerkin formulation of the collimated irradiation in frequency domain optical tomography [1]

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Abstract

In the last decade, an increasing interest has been devoted to the finite element formulations of the discrete ordinates method in solving the radiative transfer equation due to its simplicity, its flexibility, its property of being able to handle complex geometries and advection type equations. From a standard Galerkin formulation, a number of studies have been done to improve the accuracy of these models such as the Streamline Diffusion Petrov Galerkin (SUPG) [1], the Least Square formulation (LS) [2] and the Discontinuous Galerkin formulation [3], to name a few. However, optical tomography, that employs the radiative transfer equation as a forward model for light transport, involves continuous formulations of the finite elements [4, 5, 6]. However, continuous finite element formulations suffer from the lack of local conservativity compared to the discontinuous formulation which allows the use of numerical fluxes to achieve local conservativity [7]. In addition, high order accuracy can be achieved with the Discontinuous Galerkin formulation by using higher order polynomial approximation than the finite volume method. The Discontinuous finite elements formulation can be viewed as an all in one formulation [8] as with the same formulation, the finite volume method is obtained when constant polynomial elements are used , while using a continuous function yields the standard finite element method [8]. In a previous contribution [9], we have extend the inversion approach with finite elements formulations by using an integral form of the cost function that takes into account the real fact that the surface of the detectors are finite in practical application. This present paper illustrates this novel approach through a numerical study of the formulation for optical tomography purposes. The solution procedure is highlighted through a matrix-vector formulation. Numerical tests are used to gauge the accuracy of the model in both isotropic and anisotropic scattering media, with and without frequency modulation. An analysis of the collimated source direction is also worked out to show its effect on the prediction of the model.

1. To submit to the advanced thermal radiation topic.

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