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NUMERICAL SIMULATION OF THERMAL CONVECTION IN A CLOSED CAVITY IN THE PRESENCE OF A THIN HORIZONTAL HEATED PLATE

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ABSTRACT In this work, we present a numerical study of heat transfer by natural convection in a two-dimensional closed cavity, containing air, in the presence of a thin heater plate. The vertical walls are kept adiabatic, while the horizontal ones are isothermal. The equations governing the natural convection in the cavity are solved using a finite difference technique based on the control volume approach and the SIMPLEC (Semi-Implicit-Method for Pressure-Linked Equations Corrected) algorithm developed by Patankar [1980]. A non-uniform mesh in both directions, constructed by using a geometric progression, is adopted. The square cavity contains a thin heated plate located at the cavity center with an aspect ratio equal to 0.5. The heater plate is positioned horizontally and has a higher temperature than the isothermal walls. The simulation results are obtained in terms of velocity vectors and isotherms for different Rayleigh numbers values ranging from 10^4 to 10^6.
The symmetric boundary conditions produce a symmetric behaviour of temperature and velocity fields according to the central vertical plan. The increase of Rayleigh number leads to increasing importance of convection heat transfer relative to the conduction heat transfer. The fact is more marked for the regions above the heater plate. It is shown that for high Rayleigh numbers, heat transfer from the heater plate to the isothermal horizontal walls is mainly directed towards the top wall.

INTRODUCTION

Natural convection in a square cavity is relevant to many applications in cooling of electronic devices, building design, furnaces etc. The problem has been studied by several authors.
Numerical and experimental studies on natural convection in an open cavity with internal heat source were performed by Uralcan [2000]. Using FLUENT code for numerical visualization, he analyzed different locations of heat source for different Rayleigh number values. Oztop and al [2001] have conducted a numerical study of natural convection heat transfer and fluid flow in an enclosure with two heated partitions. They considered that the right side wall and the bottom wall of the enclosure were insulated perfectly while the left side wall and top wall were maintained at the same uniform temperature. Later, Oztop and al. [2004] studied natural convection in an enclosure with a heated thin plate built in vertical and horizontal location positions. They considered that upper and bottom walls of the cavity were insulated while the two vertical walls were kept at a constant temperature lower than the plate's one. They tested the effects of Rayleigh number, plate location and aspect ratio on heat transfer and fluid flow. Recently, using the same boundary conditions as Oztop and al. [2004],

Mahmoodi M. [2011] has studied free convection of a nanofluid in a square cavity with an inside heater. He performed a parametric study and investigated the effects of pertinent parameters, such as, Rayleigh number, the position and location of the heater, the volume fraction of the nanoparticles, and various types of the nanofluids on the fluid flow and heat transfer inside the cavity.

Boukhattem [2008] has conducted a numerical study, using the same computer code as in the present work, on the two dimensional heat transfer through a differentially heated square cell. The simulations were carried out for different Rayleigh numbers, and using a 42 x 42 mesh. The obtained results in terms of maximal values of components U and V of velocity respectively at half-width ($U_{\max}$) and at half-height ($V_{\max}$), the maximal stream function ($\psi_{\max}$) and the mean Nusselt number ($\overline{Nu}$) at the hot surface of the cavity were found to compare well with those of de Vahl Davis [1983] Le Breton and al. [1991] and Abdelbaki and al. [1999], figure 1.

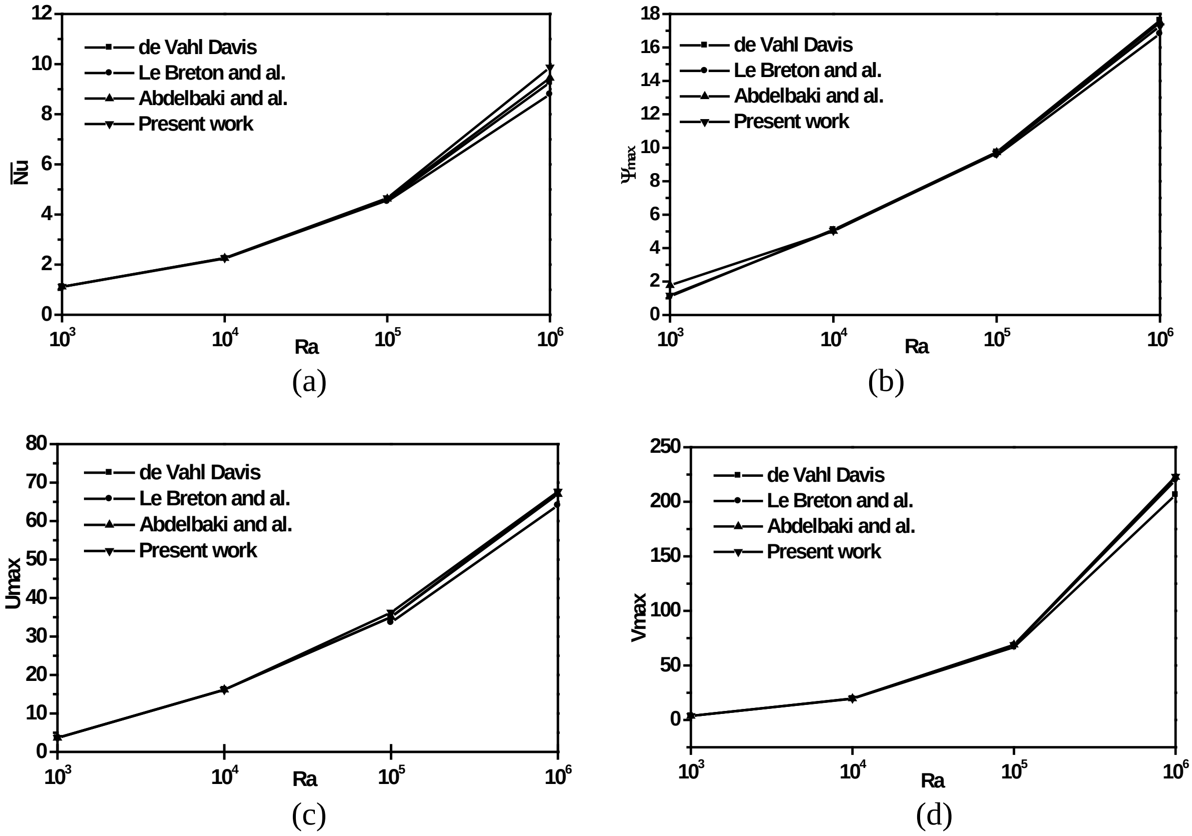


Figure 1. Comparison of the results obtained by our code and those obtained by different authors: (a) Mean Nusselt number, (b) maximal stream function, (c) U_{max} component of the velocity at mid-width (d) V_{max} component of the velocity at mid-height.

Previous authors who have studied flow and heat transfer in square cavities with heated plates, have not considered the case with adiabatic vertical walls and isothermal horizontal ones. In the present work such boundary conditions are considered. We present a study of natural convection in a square cavity with a heated thin plate built in horizontal position with an aspect ratio equal to 0.5. Boundary conditions are isothermal on the horizontal walls and adiabatic on the vertical ones. The effects of Rayleigh number on the heat transfer and fluid flow are investigated.

MATHEMATICAL MODEL

Physical Model The physical model of the problem is sketched in figure 2. It is a square cavity which contains a thin heated plate. The two vertical walls are insulated, and the temperature of the two horizontal ones is held at a uniform constant value, T_C. The heated plate is positioned horizontally in the center of the cavity and has a higher temperature, T_H, than the isothermal walls. h is the length of the plate located at the middle height, H, of the cavity. The important geometrical parameter is the aspect ratio A, defined as A=h/H.

Figure 2. Scheme of the square cavity showing boundary conditions, dimensions and position of the heated thin plate

Dimensionless Formulation of Equations and Boundary Conditions The dimensionless equations governing heat transfer and fluid flow in two dimensional laminar flow are obtained using the following variable changes:

$$\theta = \left(T - T_C\right)/(T_H - T_C),\ (X,Y) = (x,y)/H,\ (U,V) = (u,v)/(\alpha/H),\ \tau = t/(\alpha/H^2)$$

Where α is the thermal diffusivity of the fluid.

The governing equations in dimensionless form for steady flow are:

- Continuity equation:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \qquad (1)$$

- Momentum equations along X and Y:

$$\frac{\partial U}{\partial \tau} + \frac{\partial UU}{\partial X} + \frac{\partial VU}{\partial Y} = -\frac{\partial P}{\partial X} + \Pr\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) \qquad (2)$$

$$\frac{\partial V}{\partial \tau} + \frac{\partial UV}{\partial X} + \frac{\partial VV}{\partial Y} = -\frac{\partial P}{\partial Y} + \Pr\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) + \Pr R_a \theta \qquad (3)$$

- Energy equation:

$$\frac{\partial \theta}{\partial \tau} + \frac{\partial U\theta}{\partial X} + \frac{\partial V\theta}{\partial Y} = \frac{\partial \theta}{\partial X} + \frac{\partial \theta}{\partial Y} \qquad (4)$$

Where θ is the dimensionless temperature. Ra is the Rayleigh number and Pr is the Prandtl number defined respectively as : $Ra = \frac{g\beta H^3 (T_H - T_C)}{\upsilon^2} \Pr$ and $\Pr = \frac{\upsilon}{\alpha}$

The local Nusselt number is defined as:

$$Nu_x = \left(\frac{\partial T}{\partial y}\right)_w$$

The average Nusselt number is calculated by integrating the local Nusselt number along the wall:

$$\overline{Nu} = \int_0^1 Nu_x dy$$

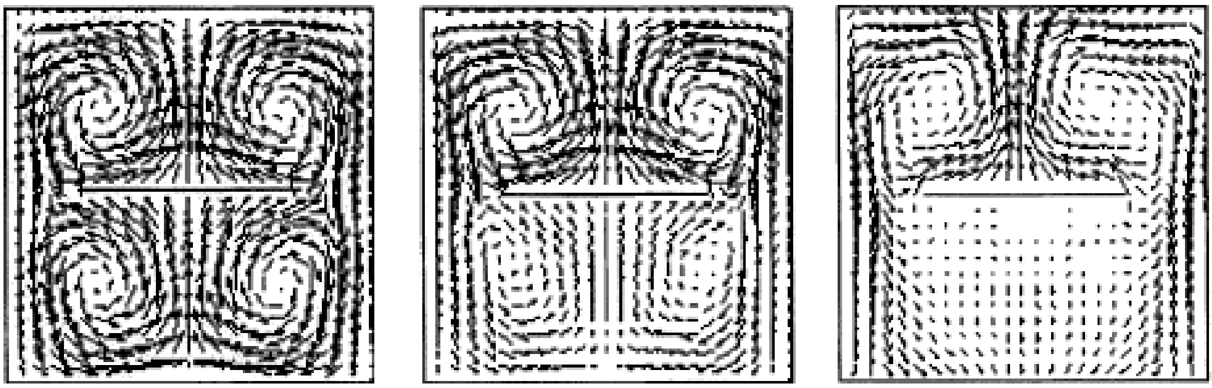
The boundary conditions of the problem are presented in dimensionless form in Table 1:

Table 1
Boundary conditions of the problem

X=0, 1	0<Y<1	U=0	V=0	$\frac{\partial T}{\partial X} = 0$
Y=0, 1	0<X<1	U=0	V=0	T=0
On the plate		U=0	V=0	T=1

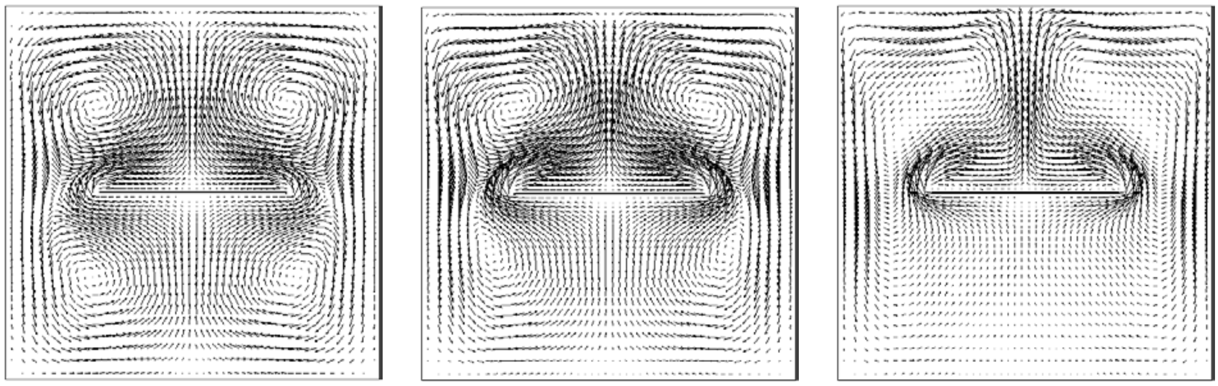
Model Validation The mathematical model was solved using a finite difference technique based on the control volume approach and the SIMPLEC algorithm developed by Patankar [1980]. The power-law difference scheme (PLDS) is used for formulation of the convection terms in the finite volume equations and central differencing (CDS) is used to discretize the diffusion terms. The finite difference equations (1–4) were solved with a line-by-line procedure using TDMA method and SIMPLEC algorithm iteratively. The final solution is checked for convergence by evaluating the error between two consecutive iteration steps ($|\varphi^k - \varphi^{k-1}| < 10^{-4}$). φ stands for u, v or T and k is the iteration number.

For model validation, we compared the results obtained by the present model and those of Oztop and al. [2004] carried out for the same geometric configuration and parameter values in the case of isothermal vertical walls and adiabatic horizontal walls. A non-uniform mesh in both directions, constructed using a geometric progression, was adopted with 50×50 grid dimension. Grid independence of solution was checked by using a 60x60 grid points and comparing values of Nusselt obtained against the case where the 50x50 grid point mesh was used. The relative variation of mean Nusselt number was found to be less than 0,38% for $Ra=10^4$ and less than 0;77% for $Ra=10^6$. The results obtained in terms of velocity vectors and isotherms are compared with those obtained by Oztop and al. [2004] for different Rayleigh numbers Ra, Figures 3 and 4. These figures show good agreement between the results obtained by our code and those of Oztop and al. [2004]. The validation of the present computer code was also checked in terms of mean Nusselt numbers for $Ra=10^4$. For A=0,4 we found $\overline{Nu} = 1{,}78$ compared to 1,51 found by Oztop and al. [2004] and for A=0,6 we found $\overline{Nu} = 2{,}35$ compared to 2,05 found by Oztop and al. [2004].



(a) (b) (c)

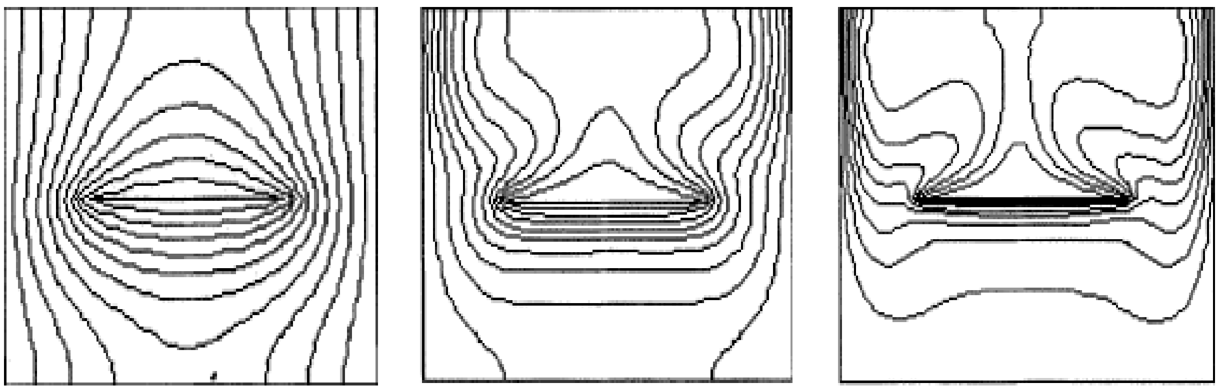
Oztop and al. [2004]



(a) (b) (c)

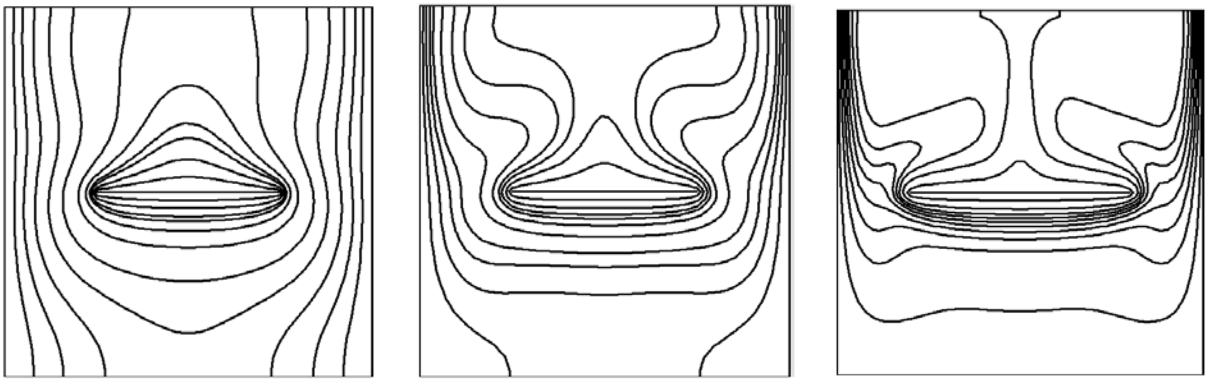
Present Work

Figure 3. Comparison of present results with those of Oztop and al. [2004]: velocity fields (a) $Ra=10^4$, (b) $Ra=10^5$, (c) $Ra=10^6$.



(a) (b) (c)

Oztop and al. [2004]



(a) (b) (c)

Present Work

Figure 4. Comparison of present results with those of Oztop and al. [2004]: isotherms (a) Ra=10^4, (b) Ra=10^5, (c) Ra=10^6.

RESULTS AND DISCUSSION

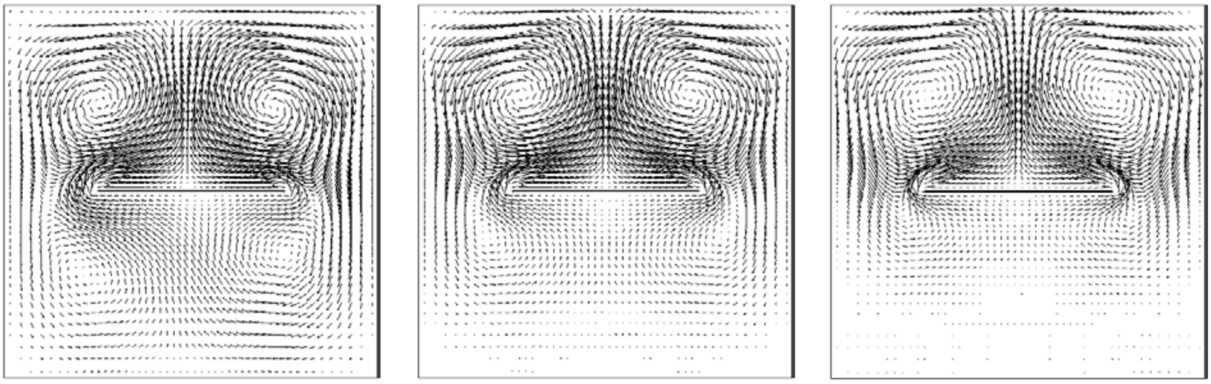
The validated model was then used to study heat transfer by natural convection in a two-dimensional closed cavity, containing air, in the presence of a thin heater plate. The vertical walls are kept adiabatic, while the horizontal ones are isothermal. The heater plate is positioned horizontally in the cavity center and has a higher temperature than the isothermal walls.

The results presented in this paper concern the study of effects of Rayleigh number, in the range 10^4 to 10^6, on velocity field and isotherms for an aspect ratio of A=0.5.

Velocity vectors Figure 5 presents velocity vectors for three Rayleigh number values : Ra=10^4, Ra=10^5 and Ra=10^6. It is shown that for Ra=10^4, four cell centers forms in the cavity; two above the heater plate and two below it. Obviously, a symmetry relative to the vertical plan located at X=0.5 exists, and cells in both sides of the heater plate rotate in the opposite direction of their symmetric ones. As the Rayleigh number increases, rotation above the plate becomes stronger compared to rotation below where stagnation points of fluid forms for higher values of Ra. This can be explained by the fact that an increase in Rayleigh number leads to an increase in the buoyancy force which pushes the cells upwards and reduces their intensities in the region below the heater plate.

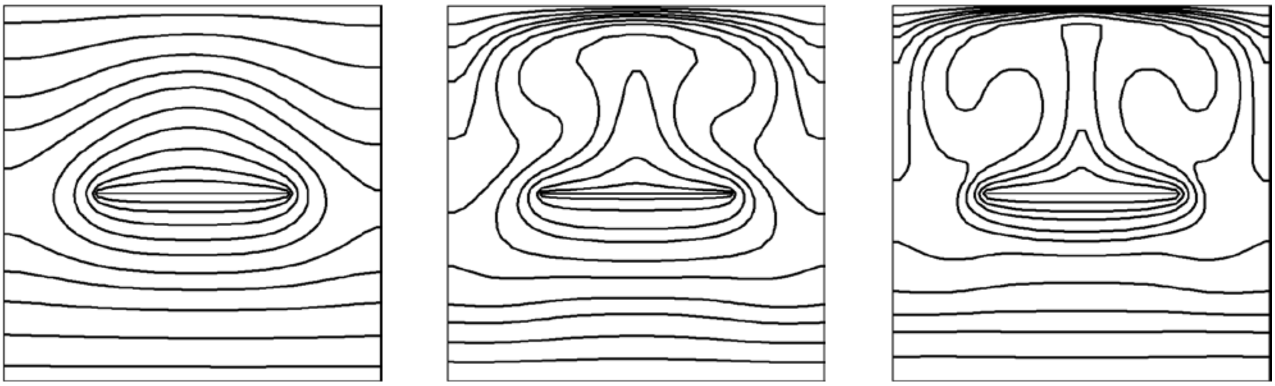
Isotherms Figure 6 displays the isotherms obtained for Rayleigh number values : Ra=10^4, Ra=10^5 and Ra=10^6. Due to symmetry relative to the vertical plan located at X=0.5, it is observed that isotherms distortion are accordingly symmetrical. For low values of Ra, convection heat transfer is negligible compared to heat transfer by conduction (figure 6 (a)), while for higher values of Ra, convection becomes important. Influence of Rayleigh number on heat transfer behavior is more marked for the region above the heater plate. The distortion of the isotherms in this region is due to the two-dimensional heat transfer. The isotherms are perpendicular to the vertical walls as adiabatic conditions are assumed there.

Intense thermal stratification occurs at the central region near the top wall for high Rayleigh values owing to the intense heat exchange with upper plate in this region favored by the flow pattern imposed by the two cells above the plate.



(a) (b) (c)

Figure 5. Velocity vectors in the cavity: (a) Ra=10^4, (b) Ra=10^5, (c) Ra=10^6.



(a) (b) (c)

Figure 6. Isotherms in the cavity : (a) Ra=10^4, (b) Ra=10^5, (c) Ra=10^6.

HEAT TRANSFER: AVERAGE AND LOCAL NUSSELT NUMBERS

AVERAGE NUSSELT NUMBER

Overall heat transfer is shown in terms of mean Nusselt numbers on the top and bottom walls of the cavity for a horizontal heated thin plate with an aspect ratio equal to 0,5. $\overline{Nu}$ is plotted as function of Ra, figure 7. As can be seen for the top wall : as Ra increases, $\overline{Nu}$ increases. For the bottom wall, opposite tendency is observed : when Ra increases, $\overline{Nu}$ decreases. For low Rayleigh number (Ra =10^4) the values of $\overline{Nu}$ are merely the same for the top and bottom walls ($\overline{Nu} \cong 1{,}7$); as Ra reaches 10^6, $\overline{Nu}$ grows up to 9,9 for the top wall, while it decreases slightly to 0,75 for the bottom wall. This shows a marked effect of natural convection on the heat transfer at the top wall while the heat transfer on the bottom one is only slightly affected.
The present work shows that for high Rayleigh numbers, heat transfer from the heater plate to the isothermal horizontal walls is mainly directed towards the top wall. This fact marks a difference with the case of the boundary conditions used in Oztop and al [2004] work where the heat transfer from the heater plate was symmetrically distributed to the vertical isothermal walls.

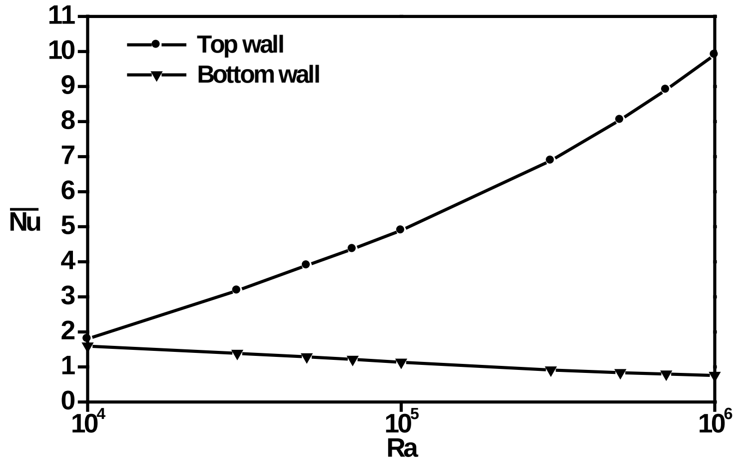


Figure 7. Evolution of $\overline{Nu}$ with Ra on the top and bottom walls of the cavity.

LOCAL NUSSELT NUMBER

Figure 8 shows the effect of Ra number on the variation of local Nusselt number with distance x at the top wall. It can be seen that, for different Ra numbers, the heat transfer or Nu(x) is low and almost the same at the edges of the top wall. It can be also observed that the heat transfer increases towards the center of the top wall with the maximum values at this center. For $Ra=10^6$, the maximum value of Nu(x) reaches Nu(0,5)=0,4.

It may be noted that the heating of the plate enhances the heat transfer at the central region of the top wall only. The physical reason for this type of behavior can be explained by the fact that at the edges, convection heat transfer is negligible compared to heat transfer by conduction because, in this region, the fluid flow is influenced by the boundary conditions (adherence conditions on the vertical walls U = 0), while on the central region, convection becomes important and favored by the two cells which are rotated in the opposite direction of their symmetric relative to the vertical plan located at X=0.5, (figure 5).

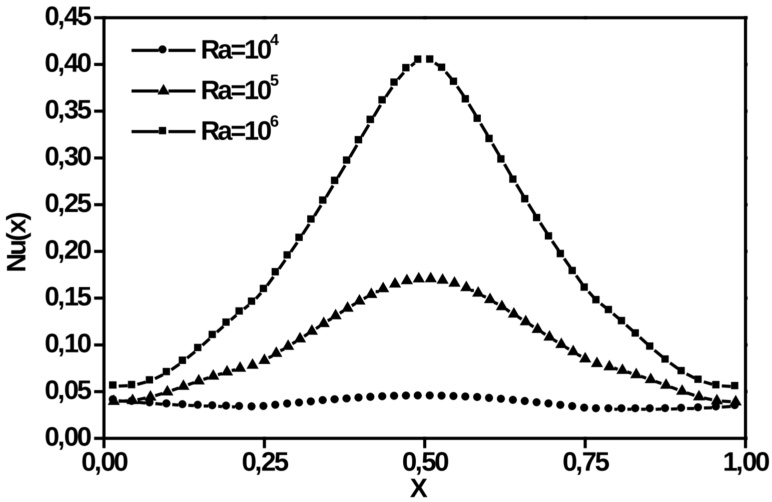


Figure 8. Variation of local Nusselt number with distance x at the top wall for different Ra number.

CONCLUSION

In this paper a model and numerical simulation of natural convection in a two-dimensional closed cavity, containing air, in the presence of a thin heater plate located at the cavity center with an aspect ratio of 0.5, were presented. Results were carried out for the case of adiabatic vertical walls and isothermal horizontal walls. The effects of Rayleigh number value, ranging from 10^4 to 10^6, on velocity and temperature fields, were studied. Heat transfer was discussed in terms of variation of mean Nusselt number on the top and bottom walls with Rayleigh number. The symmetric boundary conditions produce a symmetric behaviour of temperature and velocity fields according to the central vertical plan. The increase of Rayleigh number leads to increasing importance of convection heat transfer relative to the conduction heat transfer. The fact is more marked for the regions above the heater plate.

The present work shows that for high Rayleigh numbers, heat transfer from the heater plate to the isothermal horizontal walls is mainly directed to the top wall. This fact marks a difference with the case of the boundary conditions used in Oztop and al [2004] work where the heat transfer from the heater plate was symmetrically distributed to the vertical isothermal walls. This work will be extended to other heater plate configurations and values of aspect ratio.

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