Numerical Simulation of Thermal Convection in a Closed Cavity in the Presence of a Thin Horizontal Heated Plate

Lahcen Boukhattem [1], Hassan Hamdi [2] and Daniel. R. Rousse [3]

1. EnR2E Laboratory, CNEREE, Cady Ayyad University Marrakech, Morocco
2. LMFE CNRST Associated Research Unit (URAC27), FS, University Cady Ayyad Marrakech, Morocco
3. Industrial research chair in technologies of energy and energy efficiency (t3e), Montréal, Canada

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Abstract: In this work, a numerical study of heat transfer by natural convection in an air filled two-dimensional closed cavity in the presence of a thin heater plate is presented. The vertical walls are assumed to be adiabatic, while the horizontal ones are maintained isothermal. The equations governing the natural convection in the cavity are solved using a finite difference technique based on the control volume approach and the SIMPLEC (Semi-Implicit-Method for Pressure-Linked Equations Corrected) algorithm. A non-uniform mesh in both directions, constructed by using a geometric progression, is adopted. The square cavity contains a thin heated plate located at the cavity center with an aspect ratio equal to 0.5. The heater plate is positioned horizontally and has a higher temperature than the isothermal walls. The simulation results are obtained in terms of velocity vectors and isotherms for different Rayleigh numbers values ranging from 10^4 to 10^6.

The symmetric boundary conditions produce a symmetric behaviour of temperature and velocity fields according to the central vertical plane. The increase of Rayleigh number leads to increasing importance of convection heat transfer relative to the conduction heat transfer. The fact is more marked for the regions above the heater plate. It is shown that for high Rayleigh numbers, heat transfer from the heater plate to the isothermal horizontal walls is mainly directed towards the top wall.

Keywords: Natural convection; Square cavity; Heat transfer, thin plate.

1. Introduction

Natural convection heat transfer and fluid flow in a square cavity, in the presence of a heated plate, is relevant to many applications in cooling of electronic devices, building design, furnaces, etc. The problem, without heat source, has been studied by many researchers such as de Vahl Davis [1] who had studied a natural convection of air in a square cavity with differentially heated side walls. Natural convection in air-filled, tilted square enclosures with two adjacent walls heated and the two opposite walls cooled is numerically studied by Cianfrini et al. [2]. An experimental investigation was performed by Wu et al. [3] to examine the effect of changes in the top and bottom wall temperatures on the natural convection in an air-filled square cavity driven by a temperature difference between the vertical walls. A numerical study to investigate the steady laminar natural convection flow in a square cavity with uniformly and non-uniformly heated bottom wall, and adiabatic top wall maintaining constant temperature of cold vertical walls has been performed by Basak et al. [4].

The study of the problem of natural convection in a square cavity with a heated plate has been conducted by Dagtekin and Oztop [5].These authors have conducted a numerical study of natural convection heat transfer and fluid flow in an enclosure with two heated partitions. They considered that the right side wall and the bottom wall of the enclosure were perfectly insulated while the left side wall and top wall were maintained at the same uniform temperature. Later, Oztop et al. [6] studied natural convection in an enclosure with a heated thin plate built in vertical and horizontal location positions. They considered that upper and bottom walls of the cavity were insulated while the two vertical walls were kept at a constant temperature lower than the plate's one. They tested the effects of Rayleigh number, plate location and aspect ratio on heat transfer and fluid flow. Recently, using the same boundary conditions as Oztop et al. (2004), Mahmoodi [7] has studied free convection of a nanofluid in a square cavity with an inside heater. He performed a parametric study and investigated the effects of pertinent parameters, such as, Rayleigh number, the position and location of the heater, the volume fraction of the nanoparticles, and various types of the nanofluids on the fluid flow and heat transfer inside the cavity.

Boukhattem [8] has conducted a numerical study, using the same computer code as in the present work, on the two dimensional heat transfer through a differentially heated square cell. The simulations were carried out for different Rayleigh numbers, and using a 42 x 42 mesh. The obtained results in terms of maximal values of components U and V of velocity respectively at half-width (U_{max}) and at half-height (V_{max}), the maximal stream function (ψ_{max}) and the mean Nusselt number ($\overline{Nu}$) at the hot surface of the cavity were found to compare well with those of de Vahl Davis

Corresponding author: Lahcen Boukhattem, research fields: heat transfers, thermophysical characterization of materials. Boukhattem.lah@gmail.com

[1], Le Breton et al. [9] and Abdelbaki et Zrikem [10], Fig. 1.

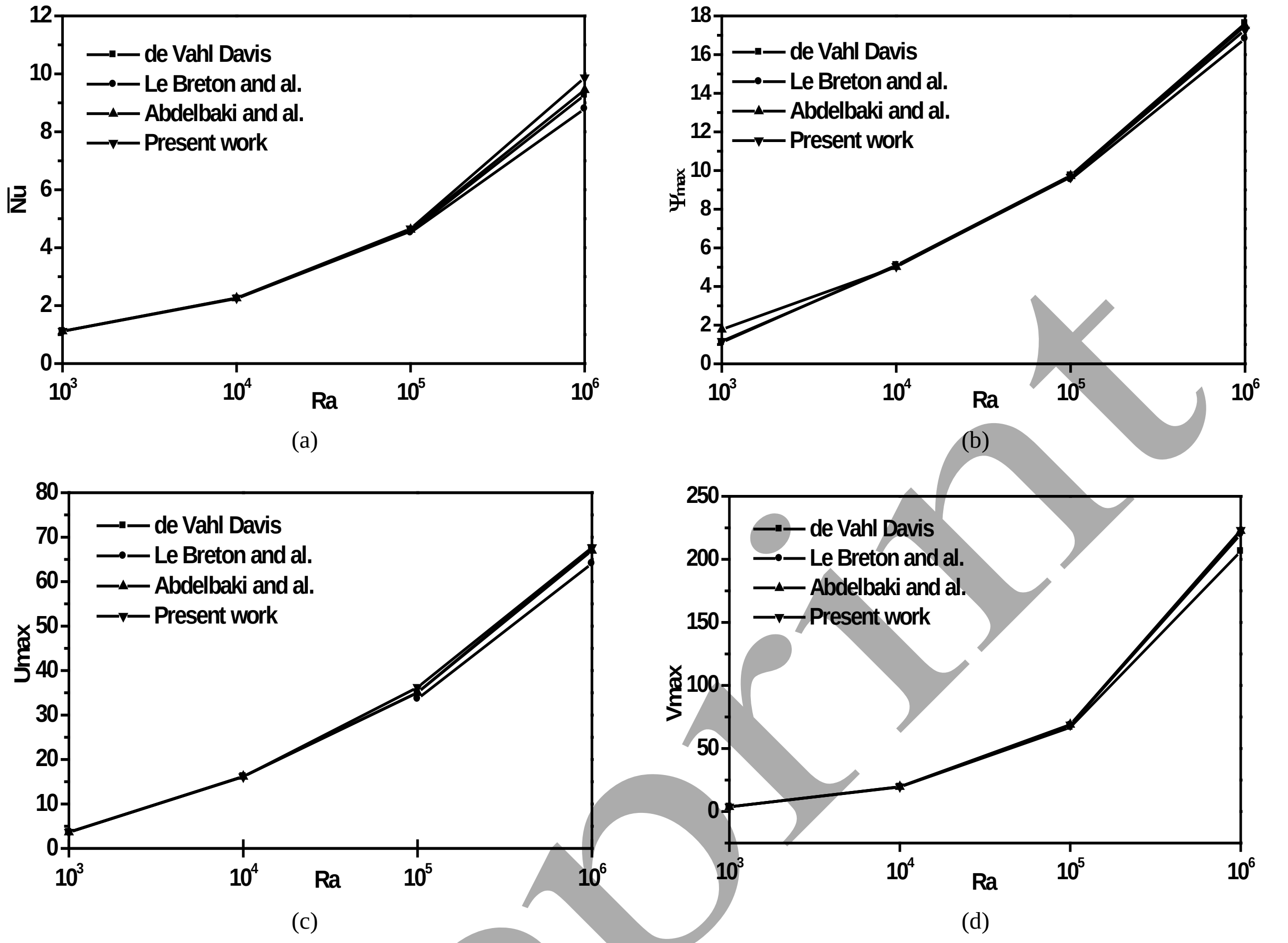


Fig. 1 Comparison of the results obtained by our code and those obtained by different authors: (a) Mean Nusselt number, (b) maximal stream function, (c) U_{max} component of the velocity at mid-width (d) V_{max} component of the velocity at mid-height.

Previous authors who have studied flow and heat transfer in square cavities with heated plates, have not considered the case with adiabatic vertical walls and isothermal horizontal ones. In the present work such boundary conditions are considered. A study of natural convection in a square cavity with a heated thin plate built in horizontal position with an aspect ratio equal to 0.5 is presented. Boundary conditions are isothermal on the horizontal walls and adiabatic on the vertical ones. The effects of Rayleigh number on the heat transfer and fluid flow are investigated.

2. Mathematical Model

2.1. Physical Model

The physical model of the problem is sketched in Fig. 2. It is a square cavity which contains a thin heated plate. The two vertical walls are insulated, and the temperature of the two horizontal ones is held at a uniform constant value, T_C. The heated plate is positioned horizontally in the center of the cavity and has a higher temperature, T_H, than the isothermal walls. h is the length of the plate located at the middle height, H, of the cavity. The important geometrical parameter is the aspect ratio A, defined as $A=h/H$.

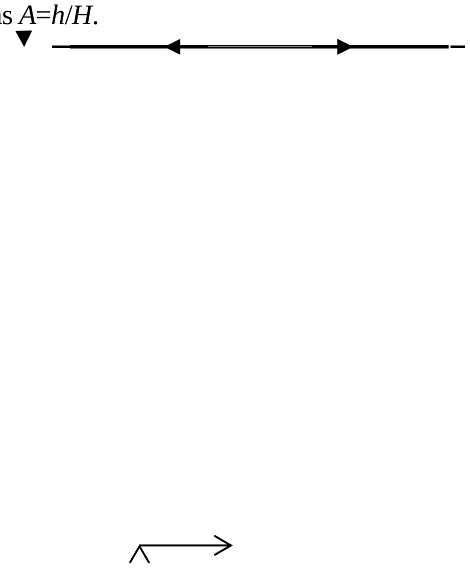


Fig. 2 Scheme of the square cavity showing boundary conditions, dimensions and position of the heated thin plate.

2. 2. Dimensionless Formulation of Equations and Boundary Conditions

The dimensionless equations governing heat transfer and fluid flow in two dimensional laminar flows are obtained using the following variable changes:

$$\theta = (T - T_C)/(T_H - T_C)\,,\ (X, Y) = (x, y)/H\,,$$

$$(U, V) = (u, v)/(\alpha / H)\,,\ \tau = t/(\alpha / H^2)$$

Where α is the thermal diffusivity of the fluid.

The governing equations in dimensionless form for steady flow are:

- Continuity equation:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \qquad (1)$$

- Momentum equations along X and Y:

$$\frac{\partial U}{\partial \tau} + \frac{\partial UU}{\partial X} + \frac{\partial VU}{\partial Y} = -\frac{\partial P}{\partial X} + \Pr\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) \qquad (2)$$

$$\frac{\partial V}{\partial \tau} + \frac{\partial UV}{\partial X} + \frac{\partial VV}{\partial Y} = -\frac{\partial P}{\partial Y} + \Pr\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) + \Pr Ra\,\theta \qquad (3)$$

- Energy equation:

$$\frac{\partial \theta}{\partial \tau} + \frac{\partial U\theta}{\partial X} + \frac{\partial V\theta}{\partial Y} = \frac{\partial \theta}{\partial X} + \frac{\partial \theta}{\partial Y} \qquad (4)$$

Where θ is the dimensionless temperature. Ra is the Rayleigh number and $\Pr$ is the Prandtl number defined respectively as : $Ra = \frac{g\beta H^3 (T_H - T_C)}{\upsilon^2} Pr$ and $Pr = \frac{\upsilon}{\alpha}$. The local Nusselt number is defined as : $Nu_x = \left(\frac{\partial T}{\partial y}\right)_w$. The average Nusselt number is calculated by integrating the local Nusselt number along the wall : $\overline{Nu} = \int_0^1 Nu_x dy.$

The boundary conditions of the problem are presented in dimensionless form in Table 1:

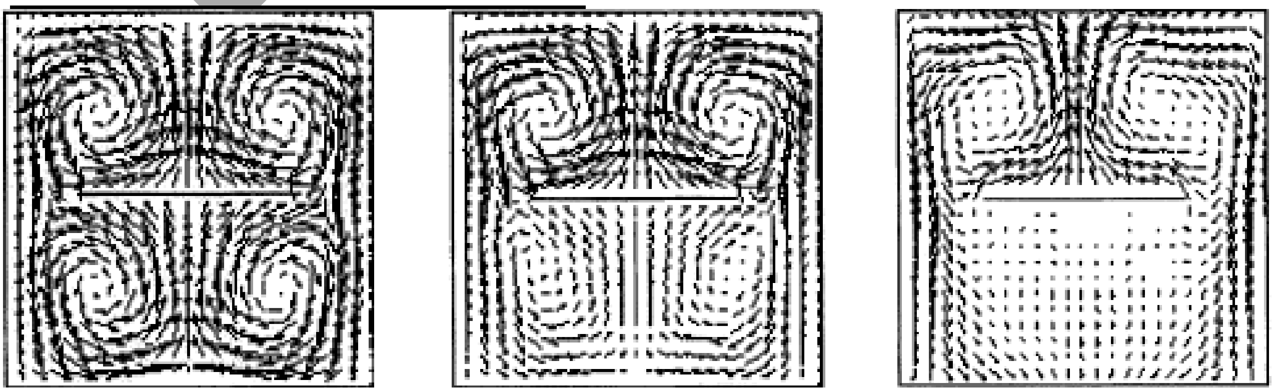
Table 1 Boundary conditions of the problem.

X=0, 1	0<Y<1	U=0	V=0	$\frac{\partial T}{\partial X} = 0$
Y=0, 1	0<X<1	U=0	V=0	T=0
On the plate		U=0	V=0	T=1

2. 3. Model Validation

The mathematical model was solved using a finite difference technique based on the control volume approach and the SIMPLEC algorithm developed by Patankar [11]. The power-law difference scheme (PLDS) is used for formulation of the convection terms in the finite volume equations and central differencing (CDS) is used to discretize the diffusion terms. The finite difference equations (1–4) were solved with a line-by-line procedure using TDMA method and SIMPLEC algorithm iteratively. The final solution is checked for convergence by evaluating the error between two consecutive iteration steps ($|\varphi^k - \varphi^{k-1}| < 10^{-4}$). φ stands for u, v or T and k is the iteration number.

For model validation, we compared the results obtained by the present model and those of Oztop et al. [6] carried out for the same geometric configuration and parameter values in the case of isothermal vertical walls and adiabatic horizontal walls. A non-uniform mesh in both directions, constructed using a geometric progression, was adopted with 50×50 grid dimension. Grid independence of solution was checked by using a 60x60 grid points and comparing values of Nusselt obtained against the case where the 50x50 grid point mesh was used. The relative variation of mean Nusselt number was found to be less than 0,38% for Ra=10^4 and less than 0;77% for Ra=10^6. The results obtained in terms of velocity vectors and isotherms are compared with those obtained by Oztop et al. [6] for different Rayleigh numbers Ra, Fig. 3 and Fig 4. These figures show a good agreement between the results proposed here and those of Oztop et al. [6].The validation of the present computer code was also checked in terms of mean Nusselt numbers for Ra=10^4. For A=0,4 we found $\overline{Nu} = 1{,}78$ compared to 1,51 found Oztop et al. [6] and for A=0,6 we found $\overline{Nu} = 2{,}35$ compared to 2,05 found by Oztop et al. [6].



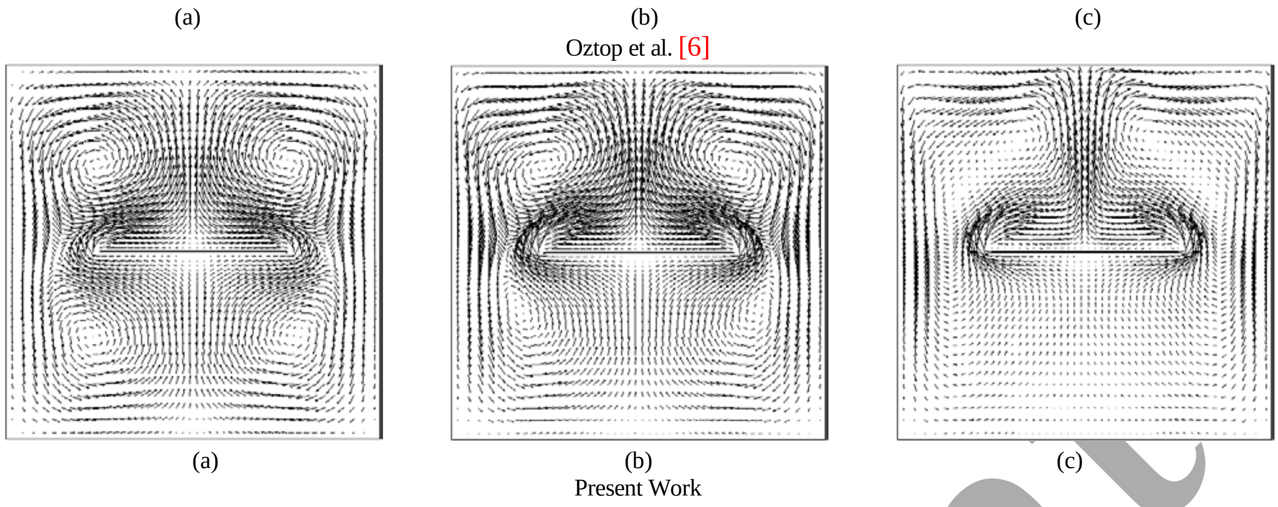


Fig 3 Comparison of present results with those of Oztop et al. [6]: velocity fields (a) $Ra=10^4$, (b) $Ra=10^5$, (c) $Ra=10^6$.

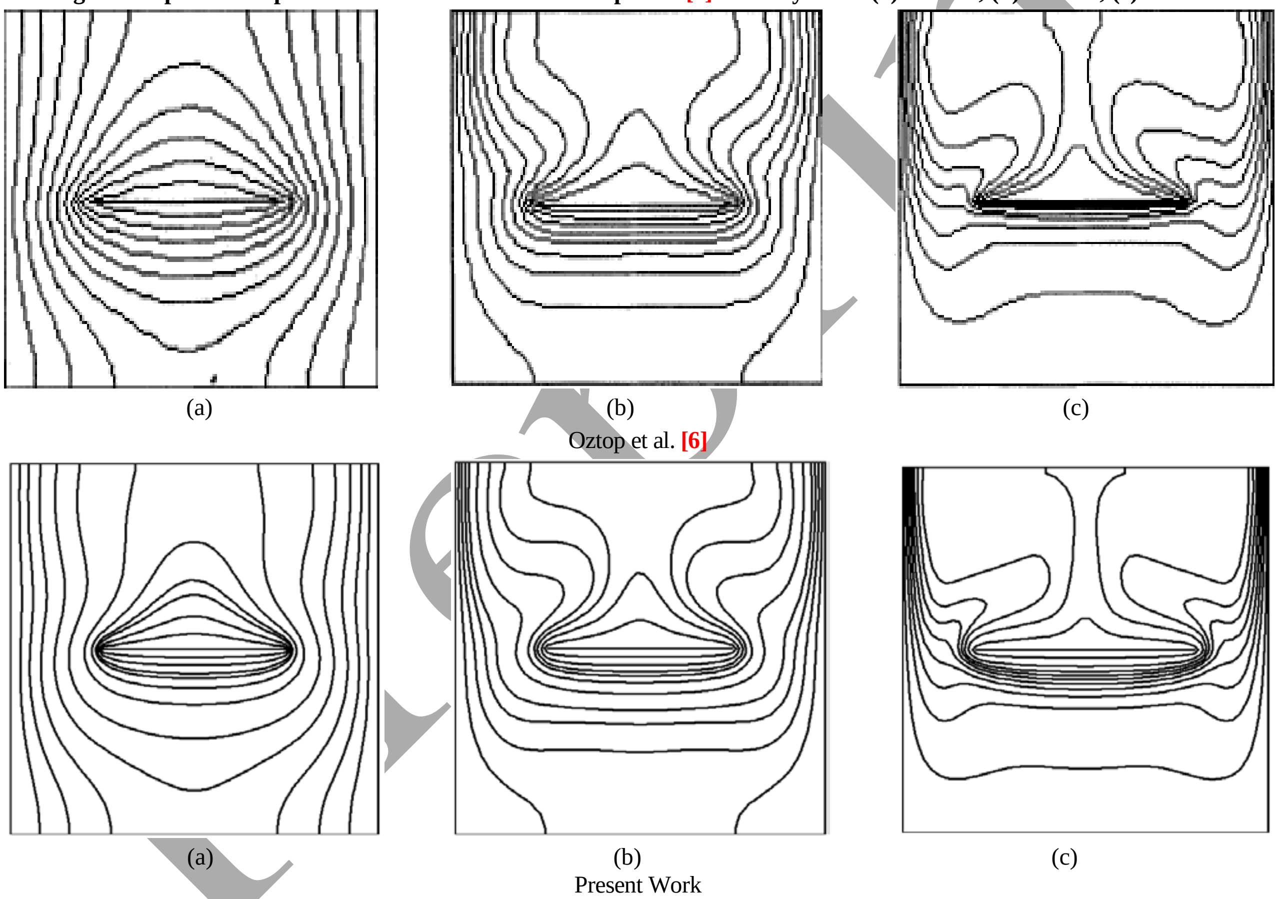


Fig.4 Comparison of present results with those of Oztop et al. [6]: isotherms (a) $Ra=10^4$, (b) $Ra=10^5$, (c) $Ra=10^6$.

3. Results and discussion

The validated model was then used to study heat transfer by natural convection in a two-dimensional closed cavity, containing air, in the presence of a thin heater plate. The vertical walls are kept adiabatic, while the horizontal ones are isothermal. The heater plate is positioned horizontally in the cavity center and has a higher temperature than the isothermal walls.

The results presented in this paper concern the study of effects of Rayleigh number, in the range 10^4 to 10^6, on velocity field and isotherms for an aspect ratio of $A=0.5$.

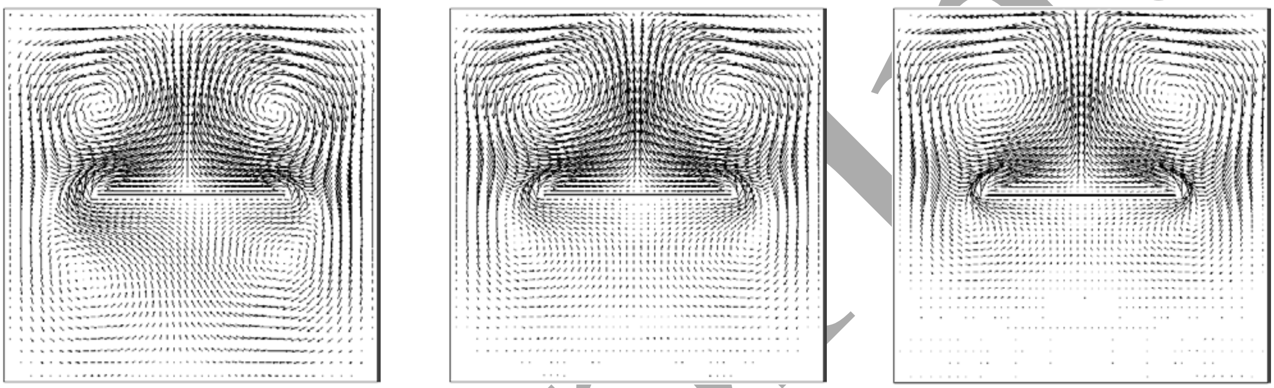
3. 1.Velocity vectors

Fig. 5 presents velocity vectors for three Rayleigh number values : $Ra=10^4$, $Ra=10^5$ and $Ra=10^6$. It is shown that for $Ra=10^4$, four cell centers forms in the cavity; two above the heater plate and two below it. Obviously, a symmetry relative to the vertical plan located at $X=0.5$ exists, and cells in both sides of the heater plate rotate in the opposite direction of their symmetric ones. As the Rayleigh number increases, rotation above the plate becomes stronger compared to rotation below where stagnation points of fluid forms for higher values of Ra. This can be explained by the fact that an increase in Rayleigh number leads to an increase in the buoyancy force which pushes the cells upwards and reduces their intensities in the region below the heater plate.

3. 2. Isotherms

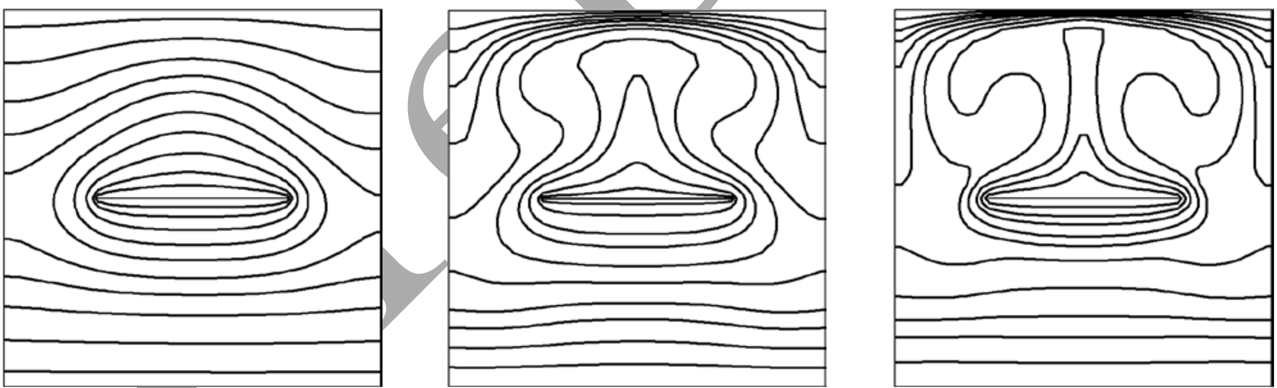
Fig. 6 displays the isotherms obtained for Rayleigh number values : $Ra=10^4$, $Ra=10^5$ and $Ra=10^6$. Due to symmetry relative to the vertical plan located at $X=0.5$, it is observed that isotherms distortion are accordingly symmetrical. For low values of Ra, convection heat transfer is negligible compared to heat transfer by conduction (Fig. 6 (a)), while for higher values of *Ra*, convection becomes important. Influence of Rayleigh number on heat transfer behavior is more marked for the region above the heater plate. The distortion of the isotherms in this region is due to the two-dimensional heat transfer. The isotherms are perpendicular to the vertical walls as adiabatic conditions are assumed there.

Intense thermal stratification occurs at the central region near the top wall for high Rayleigh values owing to the intense heat exchange with upper plate in this region favored by the flow pattern imposed by the two cells above the plate. This is confirmed by the variation of local Nusselt number at the top wall presented later (Fig. 8).



(a) (b) (c)

Fig. 5 Velocity vectors in the cavity: (a) $Ra=10^4$, (b) $Ra=10^5$, (c) $Ra=10^6$.



(a) (b) (c)

Fig. 6 Isotherms in the cavity : (a) $Ra=10^4$, (b) $Ra=10^5$, (c) $Ra=10^6$.

3. 3. Heat transfer :

a. Mean Nusselt number

The overall heat transfer variation with the Rayleigh number is shown in terms of mean Nusselt numbers on the top and bottom walls of the cavity for a horizontal heated thin plate with an aspect ratio equal to 0,5. $\overline{Nu}$ is plotted as function of *Ra*, Fig. 7. As can be seen for the top wall : as *Ra* increases, $\overline{Nu}$ increases. For the bottom wall, the opposite tendency is observed. For the lowest Rayleigh number ($Ra=10^4$) the values of $\overline{Nu}$ are merely the same for the top and bottom walls ($\overline{Nu} \cong 1,7$); as *Ra* reaches 10^6, $\overline{Nu}$ grows up to 9,9 for the top wall, while it decreases slightly to 0,75 for the bottom wall. This shows a marked effect of natural convection on the heat transfer at the top wall while the heat transfer on the bottom one is only slightly affected.

The present work shows that for high Rayleigh numbers, heat transfer from the heater plate to the isothermal horizontal walls is mainly directed towards the top wall. This fact marks a difference with the case of the boundary conditions used in Oztop et al. [6]

work where the heat transfer from the heater plate was symmetrically distributed to the vertical isothermal walls.

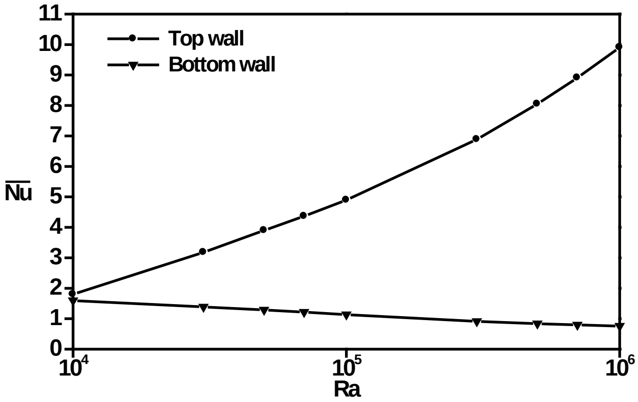


Fig. 7 Evolution of $\overline{Nu}$ with *Ra* on the top and bottom walls of the cavity.

b. Local Nusselt Number

Fig. 8 shows the variation of local Nusselt number at the top wall with position *x*, for different Rayleigh numbers. It can be seen that the variation of *Ra* has a negligible effect at the edges of the top wall and that heat transfer is very low at these positions. This is due to the fact that convection is negligible in the corners because fluid flow is influenced predominantly there by the boundary conditions (no-slip conditions on the vertical walls $U = 0$).On contrary, the variation of *Ra* has a marked effect on the heat transfer in the central region ($Nu(x)$ increases significantly with *Ra*) and the heat transfer increases from the edges to the central zone with a maximal value naturally predicted at the central location (X=0,5). For $Ra=10^6$, Nu(X=0,5)=0,4.

In the studied configuration, the heater plate enhances the heat transfer at the central region of the top wall only. Indeed in this region, convection becomes important and is favored by the two flow cells showed in Fig. 5. These cells are rotating in the opposite direction one relative to the other resulting on a cumulative effect at the center of the top wall.

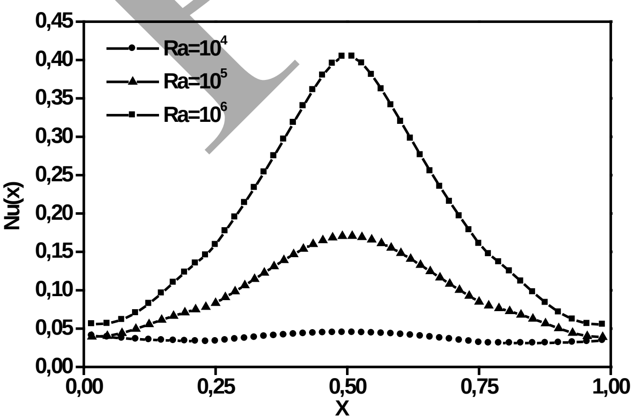


Fig. 8 Variation of local Nusselt number at the top wall with position x, for different Rayleigh numbers.

4. Conclusions

In this paper a model and numerical simulations of natural convection in an air-filled two-dimensional closed cavity in the presence of a thin heater plate located at the cavity center were presented. Validation was obtained first by comparing the proposed predictions of natural convection to results obtained by other investigators. Then, results were carried out for the case of adiabatic vertical walls and isothermal horizontal walls with a central plate involving an aspect ratio of 0.5. The effects of the variations of the Rayleigh number, ranging from 10^4 to 10^6, on velocity and temperature fields, were studied. Heat transfer was discussed in terms of variation of mean and local Nusselt numbers with the Rayleigh number. The symmetric boundary conditions was found to produce a symmetric behaviour of temperature and velocity fields according to the central vertical plane. The increase of Rayleigh number leads to increasing importance of convection heat transfer relative to the conduction heat transfer. The fact is more marked for the regions above the heater plate.

The present work shows that for high Rayleigh numbers, heat transfer from the heater plate to the isothermal horizontal walls is mainly directed to the top wall by a strong ascending movement of the heated air.. This fact marks a difference with the case of the boundary conditions used in Oztop et al. [6] work where the heat transfer from the heater plate was symmetrically distributed to the vertical isothermal walls. This work will be extended to other heater plate configurations and values of aspect ratio.

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